Electrodynamics and Radiative Processes I

Lecture 13 – Plasma effects +
Radiative processes in astrophysical systems (summary and problem solving)

Bhaswati Bhattacharyya

bhaswati@ncra.tifr.res.in

IUCAA-NCRA Graduate School

August-September 2018

Additional Ref
The Physics of Fluids and Plasma by Arnab Raichoudhuri
https://www.plasma-universe.com/

Date : 19thth September 2018

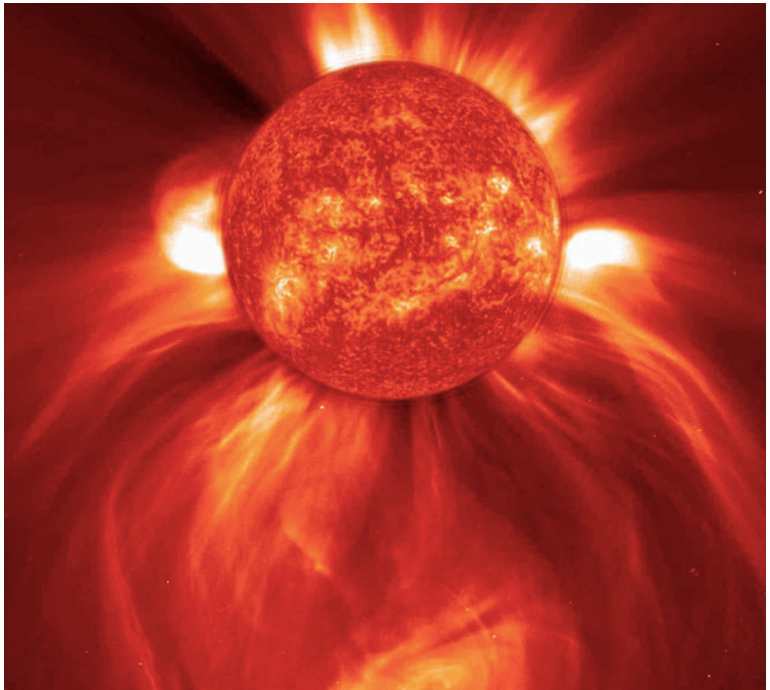
Plasma Effects

Most of the baryonic matter in the universe is plasma.

Magnetic fields play vital roles in astrophysical processes star formation, thermal conduction, accretion, turbulence, particle acceleration, dynamos, etc.

Plasma astrophysics allows the study of phenomena at extreme regions of parameter space that are inaccessible in the laboratory.

Visible Universe is 99.999% plasma.
The Sun is about 100% plasma, as are all stars.
Plasma makes up nearly 100% of the interplanetary, interstellar and intergalactic medium. The Earth's ionosphere is plasma.

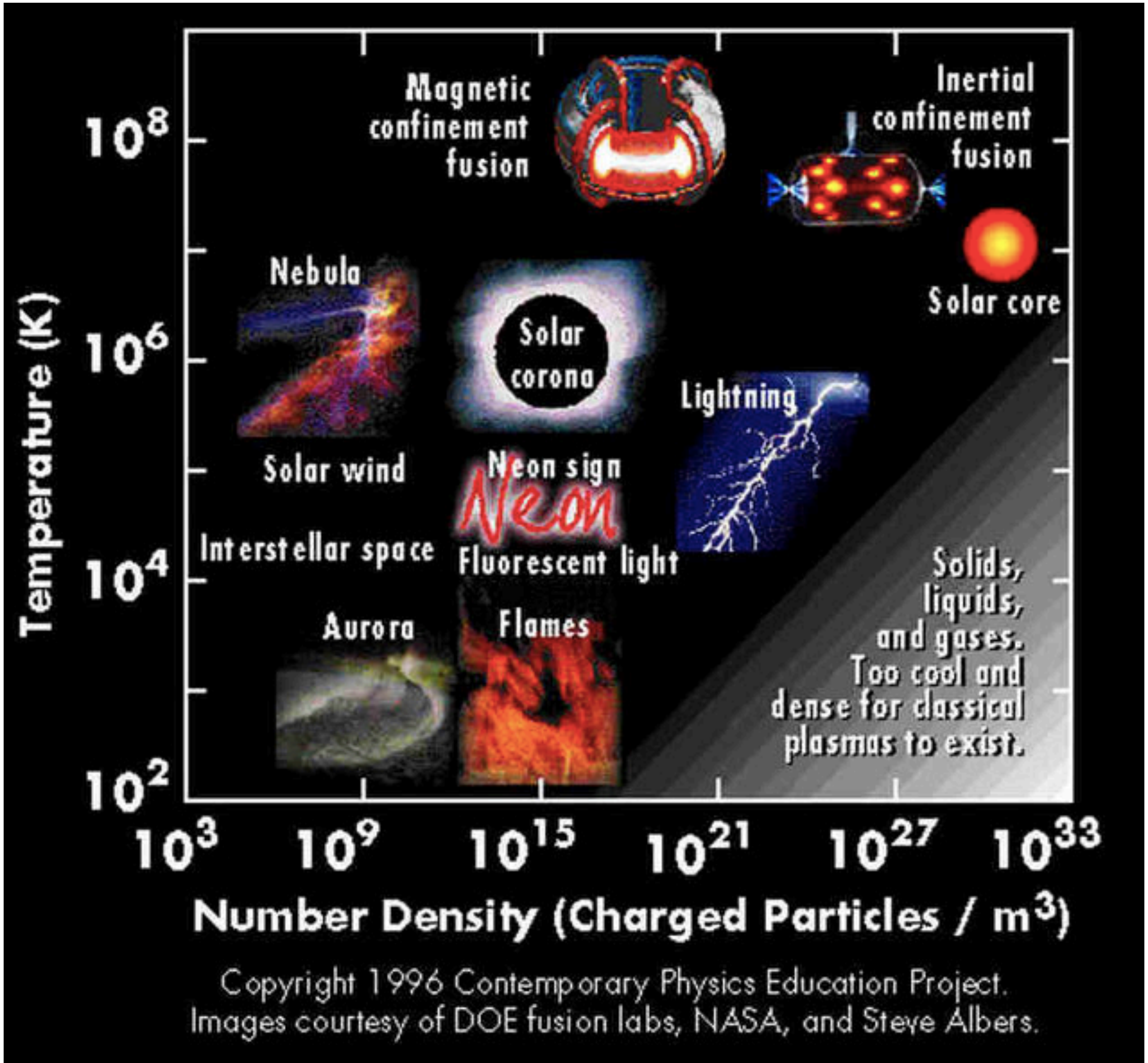


Standard definition of Plasma

- ✓ "Plasma" named by Irving Langmuir in 1920's
- ✓ The standard definition of a plasma is as the 4th state of matter (solid, liquid, gas, plasma), where the material has become so hot that (at least some) electrons are no longer bound to individual nuclei. Thus a plasma is <u>electrically conducting, and can exhibit collective dynamics.</u>
  A plasma is an ionized gas, or a partially-ionized gas (quasi-neutral).
- ✓ Even though the interaction between any pair of particles is typically weak, the collective interactions between many particles is strong. 2 examples: Debye Shielding & Plasma Oscillations.

Refer: https://www.plasma-universe.com/

Types of Plasma



The Plasma Universe is a term coined by Nobel Laureate Hannes Alfvén to highlight the importance of plasma throughout the Universe.

Plasma Effects

So far we have assumed that our propagation medium to be vacuum.

But radiation propagate through plasma.

A plasma is a gas in which an important fraction of the atoms is ionized, so that the electrons and ions are separately free.

Globally neutral ionized gas is called plasma.

Because some or all particles are electrically charged and capable of creating and interacting with electromagnetic fields, many phenomena not present in ordinary fluids and solids can be found in plasmas.

A plasma is a conductor of electricity, but a volume with dimensions greater than the so-called Debye length exhibits electrically neutral behavior. At a microscopic level, corresponding to distances shorter than the Debye length, the particles of a plasma do not exhibit collective behavior but instead react individually to a disturbance, for example, an electric field.

Plasma Astrophysics

Most of the observable matter in the Universe has been in the plasma state. In this state, normal atoms have had some or all of their electrons ripped away because of intense heating or collisions.

On the largest scales, matter is dominated by gravity, but on smaller scales, those charged ions interact with each other and with electric and magnetic fields to help create structure and channel momentum and energy.

Plasma astrophysics aims to study and help understand how plasmas behave in order to understand the detailed birth, evolution, and death of the wide variety of structures we can see in the universe: from stars and planetary systems, to galaxies and clusters of galaxies.

Reference :http://www.bu.edu/csp/files/2014/08/Zweibel_BU_2014-v2.pdf

Dispersion in cold isotropic plasma

Relation between ω and k are called dispersion relation

Consider plasma consists of electrons with density n

For a medium with dielectric constant ε → $c^2k^2 = \epsilon\omega^2$

↓

Dispersion relation

Dielectric constant ε → $\epsilon = 1 - \left(\frac{\omega_p}{\omega}\right)^2$

Plasma frequency → $\omega_p^2 = \frac{4\pi n e^2}{m}$

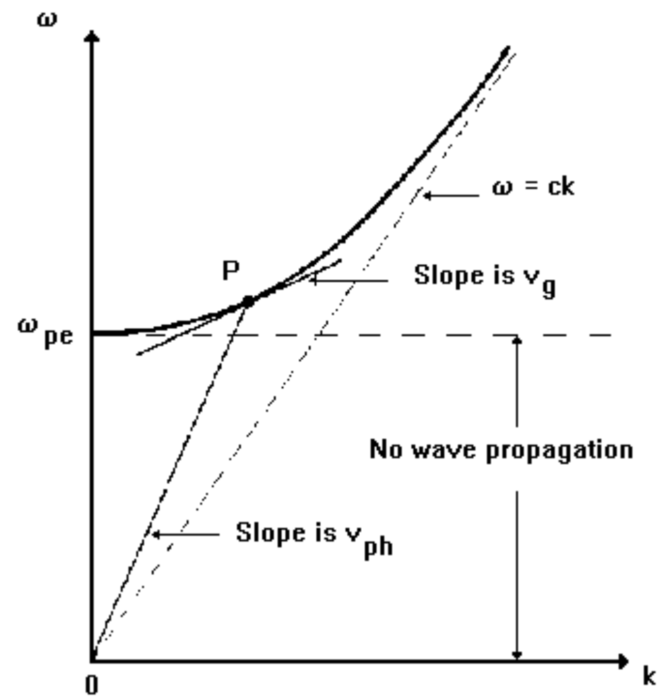
$\omega_p = 5.63 \times 10^4 n^{1/2}\, s^{-1}$, Plasma frequency for electron

Dispersion relation

All the information about the propagation of a given plasma wave mode is contained in the appropriate dispersion relation, which relates the angular frequency w to the wave number k (magnitude of the propagation vector k).

Some of the important parameters readily seen from the dispersion equation are:

(i) Phase velocity : v(ph)=w/k
(ii) Group velocity : v(g)=dw/dk
(iii) Propagation region frequency range where the wave is able to propagate
(iv) Reflection points: frequency at which the propagation region is limited by infinite phase velocity
(iv) Resonance points:
frequency at which energy can be transferred to plasma particles (zero phase velocity, and infinite group velocity)
(v) Wave growth or damping



Dispersion in cold isotropic plasma

Dispersion relation connecting k and ω can be written as

$$k = c^{-1}\sqrt{\omega^2 - \omega_p^2} \qquad \omega_p^2 = \frac{4\pi n e^2}{m}$$

$$\omega^2 = \omega_p^2 + k^2 c^2.$$

When $\omega < \omega_p$ the wave number is imaginary

ω_p : plasma cut off frequency below which no electromagnetic propagation.

Example : Earth's ionosphere prevents radiation < 1 MHz from being observed from Earth's surface (corresponding to $n \sim 10^4$ cm^{-3})

plasma frequency

Plasma frequency cutoff helps to probe ionosphere.

$$\omega_p^2 = \frac{4\pi n e^2}{m}$$

↓

Electron density can be determined as a function of height.

A pulse of radiation in a narrow range about ω be directed straight upward from Earth

When there is a layer where n is large enough to make $\omega_p > \omega$, the pulse will be totally reflected from layer.

Get information of height from time delay of pulse

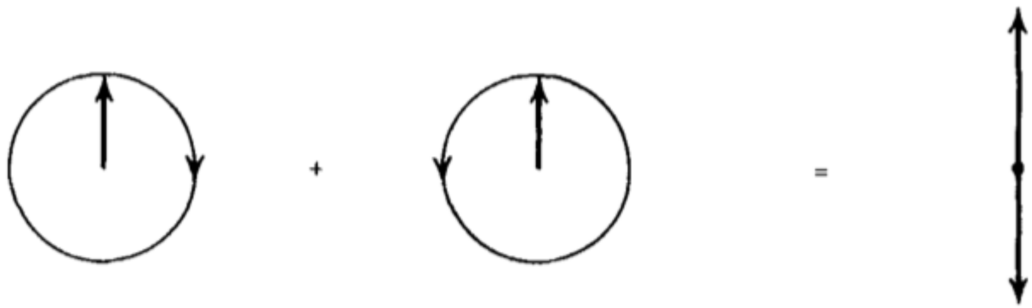
Repeating these measurements at many different frequencies electron density as a function of height can be determined

Faraday Rotation

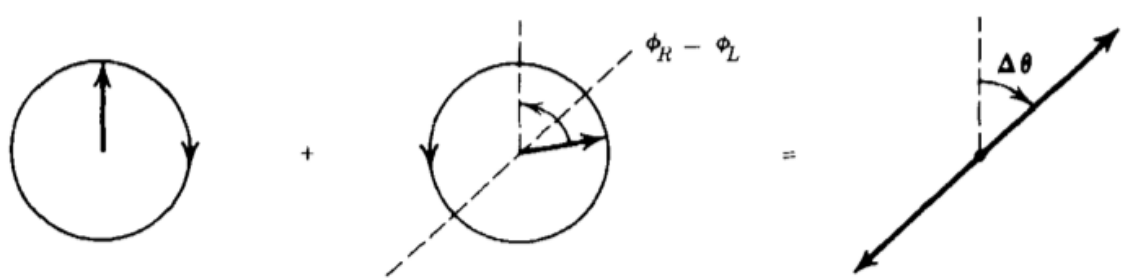
A plane polarized wave will not keep a constant plane of polarization, but its plane will rotate as it propagates.

$$\Delta\theta = \frac{2\pi e^3}{m^2 c^2 \omega^2} \int_0^d n B_{||} \, ds$$

Decomposition of linear polarization into right and left circular polarization



Faraday rotation of the plane of polarization



Fundamental processes in plasma astrophysics

- ✓ Waves
- ✓ Shocks
- ✓ Instabilities
- ✓ Turbulence
- ✓ Particle acceleration
- ✓ Dynamo : Converts kinetic energy to magnetic energy
- ✓ Reconnection: Converts magnetic energy to kinetic/thermal energy and particle energization alters magnetic field connectivity

Radiative processes at a glance
+ few problems from each topic

Radiative transfer

We can measure the following quantities:

- ✓ The energy in the radiation as a function of

a) position on the sky

b) frequency

- ✓ The radiation's polarisation.

From these measurements we can hope to determine

- ✓ Physical parameters of the source (e.g. temperature, composition, size)
- ✓ The radiation mechanism.
- ✓ The physical state of the matter.

Need to understand the difference between often used terms: luminosity, flux density, specific intensity and specific energy density.

Radiative transfer
(Lecture 1-3)

$$F_\nu = \int I_\nu \cos\theta \, d\Omega$$

$$u_\nu(\Omega) = \frac{I_\nu}{c}$$

$$dI_\nu = j_\nu \, ds \longrightarrow I_\nu(s) = I_\nu(s_0) + \int_{s_0}^{s} j_\nu(s') \, ds'$$

$$dI_\nu = -\alpha_\nu I_\nu \, ds \longrightarrow I_\nu(s) = I_\nu(s_0) \exp\left[-\int_{s_0}^{s} \alpha_\nu(s') \, ds' \right]$$

$$\alpha_\nu = n\sigma_\nu$$

$$\tau_\nu(s) = \int_{s_0}^{s} \alpha_\nu(s') \, ds'$$

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu \longrightarrow \frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

$$I_\nu(\tau_\nu) = I_\nu(0) e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

Radiative transfer

Low frequency regime

$$T_b = \frac{c^2}{2\nu^2 k} I_\nu$$

$$\frac{dT_b}{d\tau_\nu} = -T_b + T, \quad \longrightarrow \quad T_b = T_b(0)e^{-\tau_\nu} + T(1 - e^{-\tau_\nu})$$

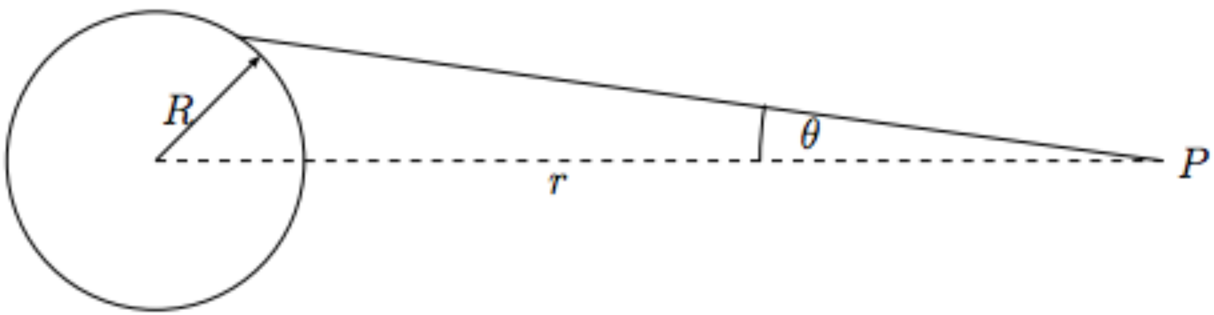
Einstein's coefficients

$$n_1 B_{12} \bar{J} = n_2 A_{21} + n_2 B_{21} \bar{J}. \quad \longrightarrow \quad g_1 B_{12} = g_2 B_{21} \qquad A_{21} = \frac{2h\nu^3}{c^2} B_{21}$$

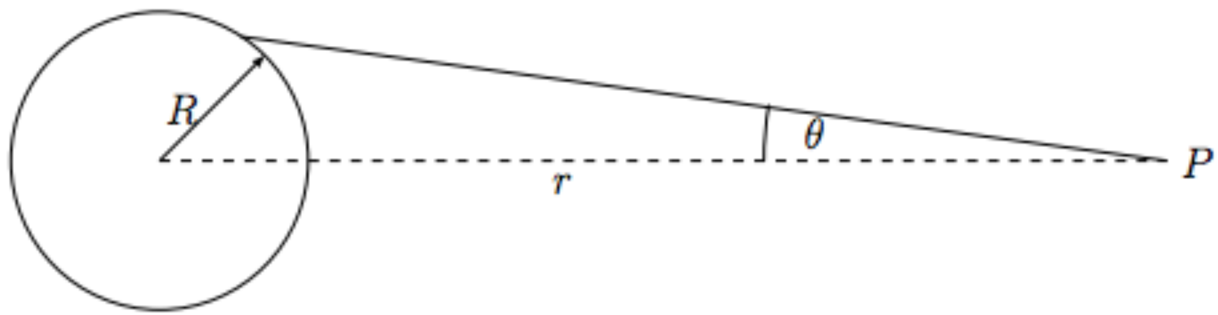
Example 1: Calculate the total flux at a point P coming from an isotropic, optically thick sphere of radius R.

Optically thick: emission comes from only surface

Isotropic: I is same for each point of the surface



Example 1: Calculate the total flux at a point P coming from an isotropic, optically thick sphere of radius R.
Optically thick: emission comes from only surface
Isotropic: I is same for each point of the surface



Solution

$$F_\nu = I_\nu \int_0^{\theta_c} \int_0^{2\pi} \cos\theta \sin\theta \, d\phi d\theta, = \pi I_\nu \sin^2\theta_c,$$

$$F_\nu = \pi I_\nu \left(\frac{R}{r}\right)^2$$

Example 2: Upon reaching a planet, part of the central star radiation will be reflected and the rest will be absorbed and re-emitted as a cooler blackbody. In the spectra of planets both of these components are observed. Consider the case of Jupiter, with radius $R_J = 7.1 \times 10^9$ cm and mean orbital radius $a_J = 7.8 \times 10^{13}$ cm. Assume that the spectrum of the Sun is a perfect blackbody.

(a) Suppose that Jupiter perfectly reflects 10% of the light coming from the Sun. Calculate its reflected luminosity. At which wavelength does it peak? In which spectral band is it observed?

(b) At which wavelength does the re-emitted luminosity peaks? In which spectral band is it observed?

Example 2: Upon reaching a planet, part of the central star radiation will be reflected and the rest will be absorbed and re-emitted as a cooler blackbody. In the spectra of planets both of these components are observed. Consider the case of Jupiter, with radius $R_J = 7.1 \times 10^9$ cm and mean orbital radius $a_J = 7.8 \times 10^{13}$ cm. Assume that the spectrum of the Sun is a perfect blackbody.

(a) Suppose that Jupiter perfectly reflects 10% of the light coming from the Sun. Calculate its reflected luminosity. At which wavelength does it peak? In which spectral band is it observed?

(b) At which wavelength does the re-emitted luminosity peaks? In which spectral band is it observed?

Hint for Solution:

(a) Solar flux reaching Jupiter $\frac{L_\odot}{4\pi a_J^2}$

Reflected Luminosity

$$L_{\rm refl} = 0.1 \times \frac{L_\odot}{4}\left(\frac{R_J}{a_J}\right)^2$$

(b) Find wave length from Wien's law $4\pi\sigma T_{\rm eff}^4 R_J^2 = 0.9 \times \frac{L_\odot}{4}\left(\frac{R_J}{a_J}\right)^2$

Basic Theory of Radiation Fields
(Lecture-4)

Maxwell's equation

$$\nabla \cdot \mathbf{D} = 4\pi\rho \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}$$

Poynting's theorem

$$\mathbf{j}\cdot\mathbf{E} + \frac{1}{8\pi}\frac{\partial}{\partial t}\left(\epsilon E^2 + \frac{B^2}{\mu}\right) = -\nabla\cdot\left(\frac{c}{4\pi}\mathbf{E}\times\mathbf{H}\right)$$

Degree of polarization

$$\Pi = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

Radiation Spectrum

$$\Delta\omega\Delta t > 1$$

Polarization and stokes parameter

Monochromatic light

$$I^2 = Q^2 + U^2 + V^2$$

$$I \equiv \mathcal{E}_1^2 + \mathcal{E}_2^2 = \mathcal{E}_0^2$$

$$Q \equiv \mathcal{E}_1^2 - \mathcal{E}_2^2 = \mathcal{E}_0^2 \cos 2\beta \cos 2\chi$$

$$U \equiv 2\mathcal{E}_1\mathcal{E}_2 \cos(\phi_1 - \phi_2) = \mathcal{E}_0^2 \cos 2\beta \sin 2\chi$$

$$V \equiv 2\mathcal{E}_1\mathcal{E}_2 \sin(\phi_1 - \phi_2) = \mathcal{E}_0^2 \sin 2\beta.$$

Radiation from moving charges (Lecture-5)

Scalar and vector potential → **E** and **B** are replaced by Φ(r,t) and **A(r,t)**

Retarded potential →

$$\phi(\mathbf{r},t)=\int\frac{[\rho]\,d^3\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}$$

$$\phi(\mathbf{r},t)=q\int\delta(t'-t+|\mathbf{r}-\mathbf{r}_0(t')|/c)\frac{dt'}{|\mathbf{r}-\mathbf{r}_0(t')|}$$

$$\phi=\left[\frac{q}{\kappa R}\right]\qquad \mathbf{A}=\left[\frac{q\mathbf{u}}{c\kappa R}\right].$$

$$\mathbf{E}(\mathbf{r},t)=q\left[\frac{(\mathbf{n}-\boldsymbol{\beta})(1-\beta^2)}{\kappa^3R^2}\right]+\frac{q}{c}\left[\frac{\mathbf{n}}{\kappa^3R}\times\{(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\}\right]$$

$$\kappa\equiv 1-\mathbf{n}\cdot\boldsymbol{\beta}.$$

$$\frac{E_{\rm rad}}{E_{\rm vel}}\sim\frac{Ruv}{c^2}=\frac{u}{c}\frac{R}{\lambda}.$$

Velocity field (near filed)

Acceleration/Radiation field (far field)

$$S=\frac{c}{4\pi}E_{\rm rad}^2=\frac{c}{4\pi}\frac{q^2\dot{u}^2}{R^2c^4}\sin^2\Theta$$

$$P=\frac{2q^2\dot{u}^2}{3c^3}$$

Larmor's Formula

An optically thin cloud surrounding a luminous object is estimated to be 1pc in radius. If the central object is clearly seen, what is an upper bound for the electron density of the cloud, assuming that the cloud is homogeneous.

An optically thin cloud surrounding a luminous object is estimated to be 1pc in radius. If the central object is clearly seen, what is an upper bound for the electron density of the cloud, assuming that the cloud is homogeneous.

Hint: $\tau = n_e \sigma_T R < 1$

Consider a particle of mass m and charge e moving ($v<<c$) in a constant magnetic field B. Show that the frequency of circular motion is $\omega_B=eB/mc$. Find out total emitted power. (RL 3.2)

Dipole approximation
(Lecture-6)

Differences in retarded time across source is negligible ⟺ $\lambda \gg L$

$$\mathbf{E}_{\rm rad} = \frac{\mathbf{n}\times(\mathbf{n}\times\ddot{\mathbf{d}})}{c^2 R_0} \iff \frac{dP}{d\Omega} = \frac{\ddot{\mathbf{d}}^2}{4\pi c^3}\sin^2\Theta \iff P = \frac{2\ddot{\mathbf{d}}^2}{3c^3}$$

Spectrum of radiation for dipole approximation

$$\frac{dW}{d\omega} = \frac{8\pi\omega^4}{3c^3}|\hat{d}(\omega)|^2$$

$$\mathbf{d} = -\left(\frac{e^2 E_0}{m\omega_0^2}\right)\boldsymbol{\epsilon}\sin\omega_0 t$$

Thomson scattering cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm polarized} = \frac{e^4}{m^2c^4}\sin^2\Theta = r_0^2\sin^2\Theta$$

$$\sigma = \int \frac{d\sigma}{d\Omega}d\Omega = 2\pi r_0^2\int_{-1}^{1}(1-\mu^2)d\mu = \frac{8\pi}{3}r_0^2 = \sigma_T \sim 0.66\times10^{-24}\ \mathrm{cm}^2$$

Relativity
(Lecture 6,7)

Special theory of relativity

Length contraction (length of a moving rod appears smaller) $L=\left(1-\frac{v^2}{c^2}\right)^{1/2}L_0$

Time dilation (moving clock appears slower) $T=t_2-t_1=\gamma(t_2'-t_1')=\gamma T_0$

Transformation of velocities $u_{\parallel}=\frac{u_{\parallel}'+v}{\left(1+vu_{\parallel}'/c^2\right)},\qquad u_{\perp}=\frac{u_{\perp}'}{\gamma\left(1+vu_{\parallel}'/c^2\right)}$

Addition of velocities

Beaming effect $\theta\sim\frac{1}{\gamma}$

Energy of a moving body $E_k=m_oc^2/\surd(1+v^2/c^2)$

Relativistic Doppler effect $\omega=\frac{2\pi}{\Delta t_A}=\frac{\omega'}{\gamma\left(1-\frac{v}{c}\cos\theta\right)}$

Proper time $c^2d\tau^2=c^2dt^2-\left(dx^2+dy^2+dz^2\right)$

Four vectors

Space-time is a four-vector: x^μ = [ct, x]
For μ=0,1,2,3

Problem based on these relations

Relativity

Examples of four vectors

Space-time is a four-vector: $x^{\mu} = [ct, \mathbf{x}]$

Four-vectors have Lorentz transformations between two frames with uniform relative velocity v:

$$x' = \gamma(x - \beta ct); \qquad ct' = \gamma(ct - \beta x)$$

$$x^{\mu}x^{\nu} = c^2t^2 - |x|^2 = c^2t'^2 - |x'|^2 = s^2$$

Charge/current four-vector $J^{\mu} = [c\rho, \mathbf{J}]$

Potential four-vector $A^{\mu} = \left[\frac{V}{c}, \mathbf{A}\right]$

Continuity equation, Lorentz gauge condition, Poisson's equations in terms of four vectors

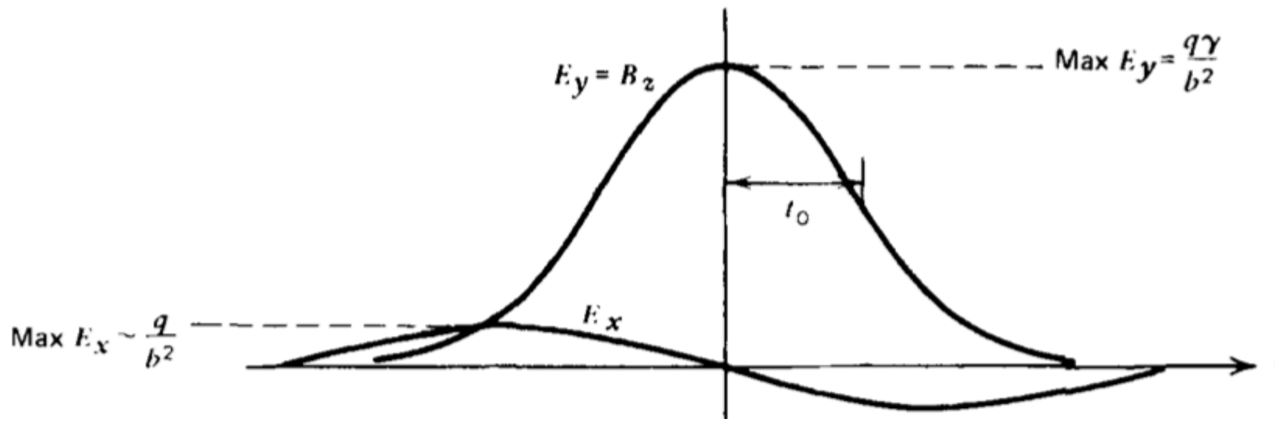
Electromagnetic field tensor

$$F^{\mu\nu} = \frac{\partial A_{\nu}}{\partial x^{\mu}} - \frac{\partial A_{\mu}}{\partial x^{\nu}}$$

Express Maxwell's equations in terms of electromagnetic field tensor

Relativity

Method of virtual quanta



Fields are mostly transverse (in y direction) since (Max E_x)/(Max E_y)= Y

Fields of the moving charges are concentrated in the plane transverse to its motion into an angle of order of 1/γ

$$P = \frac{dW}{dt}, \qquad P' = \frac{dW'}{dt'}$$

Emitted power is Lorentz invariant

$$P = \frac{2q^2}{3c^3}\mathbf{a}'\cdot\mathbf{a}' = \frac{2q^2}{3c^3}\left(a_{\parallel}'^2 + a_{\perp}'^2\right) = \frac{2q^2}{3c^3}\gamma^4\left(a_{\perp}^2 + \gamma^2 a_{\parallel}^2\right)$$

Relativity

Example 1: A source has a specific intensity I_ν and is moving relativistically with respect to a stationary observer (in the reference frame K). Derive the relationship between the specific intensity measured by an observer in the reference frame K and the specific intensity seen in the comoving frame K_0 (i.e., the frame where the source is at rest).

Relativity

Example 1: A source has a specific intensity I_ν and is moving relativistically with respect to a stationary observer (in the reference frame K). Derive the relationship between the specific intensity measured by an observer in the reference frame K and the specific intensity seen in the comoving frame K_0 (i.e., the frame where the source is at rest).

Hint:

$$I(\nu) = \frac{\mathrm{d}E}{\mathrm{d}t\mathrm{d}\nu\mathrm{d}\Omega\mathrm{d}A}$$

Hint : $dE=h\nu dN= \gamma dE'$

$d\Omega=\gamma^2\, d\Omega'$

$I_\nu/\gamma 3$ =Lorentz invarient

Relativity

In non relativistic domain, a source of radiation having fluctuation of duration Δt must have a physical diameter of the source of order of $D<c\Delta t$.
Consider an optically thick spherical shell of radius $R(t)$ that is expanding with relativistic velocity $\beta \sim 1$, $\gamma >> 1$, and is energized by a stationary point source at its center.
Considering relativistic beaming effects show that if the observer sees a fluctuation from the shell of duration Δt at time t , the source may actually be of radius $R<2\gamma^2 c\Delta t$.
(RL 4.1)

Relativity

In non relativistic domain, a source of radiation having fluctuation of duration Δt must have a physical diameter of the source of order of $D<c\Delta t$.
Consider an optically thick spherical shell of radius $R(t)$ that is expanding with relativistic velocity $\beta \sim 1$, $\gamma >> 1$, and is energized by a stationary point source at its center.
Considering relativistic beaming effects show that if the observer sees a fluctuation from the shell of duration Δt at time t , the source may actually be of radius $R<2\gamma^2 c\Delta t$.
(RL 4.1)

Hint: $\Delta t > R/c(1-\cos\theta_c) \sim (R/2c)\,\theta c^2$, $\theta c \sim 1/\gamma$

Bremsstrahlung
(Lecture 8,9)

Emissivity (energy emitted per unit volume per unit time per unit frequency)

$$\frac{dW}{dV\,dt\,d\nu} = \frac{2^5\pi e^6}{3mc^3}\left(\frac{2\pi}{3km}\right)^{1/2} T^{-1/2} Z^2 n_e n_i e^{-h\nu/kT}\bar{g}_{ff}$$

$$\varepsilon_\nu^{ff} \equiv \frac{dW}{dV\,dt\,d\nu} = 6.8\times 10^{-38} Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT}\bar{g}_{ff}$$

$$\varepsilon^{ff} \equiv \frac{dW}{dt\,dV} = 1.4\times 10^{-27} T^{1/2} n_e n_i Z^2 \bar{g}_B$$

Free-free absorption coefficient

$$\alpha_\nu^{ff} = \frac{4e^6}{3mhc}\left(\frac{2\pi}{3km}\right)^{1/2} T^{-1/2} Z^2 n_e n_i \nu^{-3}\left(1-e^{-h\nu/kT}\right)\bar{g}_{ff}$$

$$\alpha_\nu^{ff} = 3.7\times 10^{8} T^{-1/2} Z^2 n_e n_i \nu^{-3}\left(1-e^{-h\nu/kT}\right)\bar{g}_{ff}$$

Bremsstrahlung

- ✓ At low ν, $\tau_\nu >> 1$ → Black body like spectrum
- ✓ At high ν, $\tau_\nu << 1$ → Flat spectrum

Turn over at $h\nu = kT$

Non thermal Bremsstrahlung

$$\frac{dW}{dV\,dt} = 1.4 \times 10^{-27} T^{1/2} Z^2 n_e n_i \bar{g}_B (1 + 4.4 \times 10^{-10} T)$$

Bremsstrahlung

Example 1: A ionized hydrogen gas ($Z = 1$) of a number density $ne = ni = 6 \times 10^2$ cm^{-3} of a size 10^{19} cm and initial temperature of 800 0K cools via thermal bremsstrahlung.

(a) How long does it take for the gas to cool down to $T = 0$? Assume Gaunt factor $g = 1$.

(b) Find the luminosity of the entire nebula in terms of solar luminosities.

Bremsstrahlung

A ionized hydrogen gas (Z = 1) of a number density ne = ni = 6 × 10^2 cm^{-3} of a size 10^{19} cm and initial temperature of 800 0K cools via thermal bremsstrahlung.

(a) How long does it take for the gas to cool down to T = 0? Assume Gaunt factor g = 1.

(b) Find the luminosity of the entire nebula in terms of solar luminosities.

Hint: $L = \varepsilon_{ff} V$

$$\textbf{Cooling Time } T_{cool} = \frac{\text{Energy content of a gas}}{\text{Rate at which energy is being radiated}}$$

Example 2: Orion nebula is one of the the brightest HII regions on the sky. Its angular size is approximately 1 deg and we know that its distance is around 400 pc. This region emits thermal bremsstrahlung radiation with transition from optically thick to optically thin regime at 1 GHZ and cut-off frequency at 200 THz.

(a) In observations, the measure of the cutoff frequency is a way to determine the plasma temperature. Estimate the temperature of Orion nebula.

(b) Assuming that NGC1976 has a spherical shape, estimate the number density of the region

Example 2: Orion nebula is one of the the brightest HII regions on the sky. Its angular size is approximately 1 deg and we know that its distance is around 400 pc. This region emits thermal bremsstrahlung radiation with transition from optically thick to optically thin regime at 1 GHZ and cut-off frequency at 200 THz.

(a) In observations, the measure of the cutoff frequency is a way to determine the plasma temperature. Estimate the temperature of NGC1976.

(b) Assuming that NGC1976 has a spherical shape, estimate the number density of the region

Hint: a) cut-off frequency $h\nu = kT$, hence determine temperature

b) Optically thin→optically thick at $\tau=1$, $\tau=\alpha L$

Example 3: Consider a sphere of ionized hydrogen plasma that is undergoing spherical gravitational collapse. The sphere is held at constant isothermal temperature T_0, uniform density and constant mass M_0 during the collapse, and has decreasing radius R(t). The sphere cools by emission of bremsstrahlung radiation in its interior. At $t = t_0$ the sphere is optically thin.

(a) What is the total luminosity of the sphere as a function of M_0, R(t) and T_0 while the sphere is optically thin

(b) What is the luminosity of the sphere as a function of time after it becomes optically thick?

(c) Give an implicit relation, in terms of R(t), for the time t1 when the sphere becomes optically thick.

(d) Draw a qualitative curve of the luminosity as a function of time.

Example 3: Consider a sphere of ionized hydrogen plasma that is undergoing spherical gravitational collapse. The sphere is held at constant isothermal temperature T_0, uniform density and constant mass M_0 during the collapse,
and has decreasing radius R(t). The sphere cools by emission of bremsstrahlung radiation in its interior. At $t = t_0$ the sphere is optically thin.
(a) What is the total luminosity of the sphere as a function of M_0, R(t) and T_0 while the sphere is optically

(b) What is the luminosity of the sphere as a function of time after it becomes optically thick?
(c) Give an implicit relation, in terms of R(t), for the time t1 when the sphere becomes optically thick.
(d) Draw a qualitative curve of the luminosity as a function of time.

Hint: (a) $L_{thin} = \varepsilon_{ff} V$

(b) $L_{thick} = 4\pi R^2 \sigma T^4$

(c) $L_{thick} = L_{thin}$ find out relation between $R(t_0)$ versus T_0

Synchrotron + Compton
(Lectures 10,11,12)

Problems on cooling time calculation.

Calculate characteristic synchrotron frequency for electrons with Y~10^4 at a magnetic field of 0.1 G. What is the frequency of gyration? Show that in this case photon energy in electrons rest frame is small compared to mc^2. What does this imply?

Synchrotron + Compton

Problems on cooling time calculation.

Calculate characteristic synchrotron frequency for electrons with Y~10^4 at a magnetic field of 0.1 G. What is the frequency of gyration? Show that in this case photon energy in electrons rest frame is small compared to mc^2.

Synchrotron

What is the typical energy of a scattered photon when 2 GeV cosmic ray electrons interact with the photons of the microwave background radiation, which has a temperature of T = 2.73K. What is the Lorentz factor of incident electrons?

Hint: $\epsilon_1 = 4/3\gamma^2\epsilon.$

$E_e = \gamma m_e c^2$

↓ Black body

↓ Determine γ

About Final Exam

27th August(2:30-5:30)

Closed book/notes
Calculators allowed

Total :100 marks

Objective: 10 Theory: 30 Problem: 60

Request

Explain all the symbols you use.
Complete numerical calculations.
Write neatly.

Practice problems discussed in Lectures, Assignment, Mini-tests, Rybicki and Lightman and more

End of Lectures

Thank you