

# Electrodynamics and Radiative Processes I

## Lecture 12 – Synchrotron self-absorption Compton/Inverse-Compton scattering

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Reference :

1) Rybicki and Lightman

2) Ghisellini: <http://www.brera.inaf.it/utenti/gabriele/total.pdf>

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# Synchrotron self-absorption(Recap)

Synchrotron emission process is accompanied by absorption in which

- a) A photon interacts with a charge in magnetic field and is absorbed giving up its energy to the charge
- b) Stimulated emission (or negative absorption) in which a particle is induced to emit more strongly into a direction and at a frequency where photons are already present

These processes are related by Einstein's coefficient

$$\alpha_\nu = \frac{h\nu}{4\pi} \sum_{E_1} \sum_{E_2} [n(E_1)B_{12} - n(E_2)B_{21}] \phi_{21}(\nu)$$

$\phi_{21}(\nu)$  is  $\delta$  function that restricts summations to these states differing by an energy  $h\nu = E_2 - E_1$

# Synchrotron self-absorption(Recap)

$$\alpha_\nu = \frac{h\nu}{4\pi} \sum_{E_1} \sum_{E_2} [n(E_1)B_{12} - n(E_2)B_{21}] \phi_{21}(\nu)$$

Now we want to write the absorption coefficient so that it contain the expression of power which we discussed,

$$P(\omega) = \frac{\sqrt{3}}{2\pi} \frac{q^3 B \sin \alpha}{mc^2} F\left(\frac{\omega}{\omega_c}\right)$$

It is convenient to write the emission in terms of the frequency  $\nu$  rather than  $\omega$ .  
So we use  $P(\nu, E_2) = 2\pi P(\omega)$ .

# Synchrotron self-absorption(Recap)

$$\alpha_\nu = \frac{h\nu}{4\pi} \sum_{E_1} \sum_{E_2} [n(E_1)B_{12} - n(E_2)B_{21}] \phi_{21}(\nu)$$

It is convenient to write the emission in terms of the frequency  $\nu$  rather than  $\omega$ .  
So we use  $P(\nu, E_2) = 2\pi P(\omega)$ .

Relations between Einstein's coefficients

$$g_1 B_{12} = g_2 B_{21},$$
$$A_{21} = \frac{2h\nu^3}{c^2} B_{21}$$

Total power emitted per frequency of a single particle can be written as

$$P(\nu, E_2) = h\nu \sum_{E_1} A_{21} \phi_{21}(\nu)$$
$$= (2h\nu^3 / c^2) h\nu \sum_{E_1} B_{21} \phi_{21}(\nu)$$

This expression relates the spontaneous emission ( $A_{21}$ ) with the stimulated emission ( $B_{21}$ )

# Synchrotron self-absorption(Recap)

$$\alpha_\nu = \frac{h\nu}{4\pi} \sum_{E_1} \sum_{E_2} [n(E_1)B_{12} - n(E_2)B_{21}] \phi_{21}(\nu).$$

Absorption coefficient due to stimulated emission

$$\frac{-h\nu}{4\pi} \sum_{E_1} \sum_{E_2} n(E_2)B_{21}\phi_{21} = \frac{-c^2}{8\pi h\nu^3} \sum_{E_2} n(E_2)P(\nu, E_2).$$

Absorption coefficient due to true absorption

$$\frac{h\nu}{4\pi} \sum_{E_1} \sum_{E_2} n(E_1)B_{12}\phi_{21} = \frac{c^2}{8\pi h\nu^3} \sum_{E_2} n(E_2 - h\nu)P(\nu, E_2)$$

$$\alpha_\nu = \frac{c^2}{8\pi h\nu^3} \sum_{E_2} [n(E_2 - h\nu) - n(E_2)] P(\nu, E_2).$$

# Synchrotron self-absorption(Recap)

Consider isotropic electron distribution function  $f(p)$

$f(p) d^3p$  = number of electrons per unit volume with momentum in  $d^3p$  about  $p$

$$= \tilde{\omega} h^{-3} d^3p,$$



(statistical weight of the particle, it has nothing to do with angular frequency; for electrons it's 2 (spin up/spin down states))

So we can make the substitution  $\sum_2 \rightarrow \frac{\tilde{\omega}}{h^3} \int d^3p_2,$   $n(E_2) \rightarrow \frac{h^3}{\tilde{\omega}} f(p_2).$

# Synchrotron self-absorption(Recap)

$$\alpha_\nu = \frac{c^2}{8\pi h\nu^3} \sum_{E_2} [n(E_2 - h\nu) - n(E_2)] P(\nu, E_2) \quad + \quad \sum_2 \rightarrow \frac{\tilde{\omega}}{h^3} \int d^3p_2, \quad n(E_2) \rightarrow \frac{h^3}{\tilde{\omega}} f(p_2).$$

So the absorption coefficient is

$$\alpha_\nu = \frac{c^2}{8\pi h\nu^3} \int d^3p_2 [f(p_2^*) - f(p_2)] P(\nu, E_2)$$

where  $p_2^*$  is the momentum corresponding to energy  $E_2 - h\nu$

Check if this formula produces correct results for thermal distribution of particles

$$f(p) = K \exp\left[-\frac{E(p)}{kT}\right].$$

# Synchrotron self-absorption (Recap)

$$f(p_2^*) - f(p_2) = K \exp\left(-\frac{E_2 - h\nu}{kT}\right) - K \exp\left(-\frac{E_2}{kT}\right)$$

$$= f(p_2)(e^{h\nu/kT} - 1).$$

The absorption coefficient is

$$(\alpha_\nu)_{\text{thermal}} = \frac{c^2}{8\pi h\nu^3} (e^{h\nu/kT} - 1) \int d^3p_2 f(p_2) P(\nu, E_2)$$

$$4\pi B_\nu(T)$$

total power per unit volume  
per unit frequency range  $4\pi j_\nu$

$$(\alpha_\nu)_{\text{thermal}} = \frac{j_\nu}{B_\nu(T)}$$

Correct result for thermal emission  
Kirchhoff's Law

# Synchrotron self-absorption

The next step is to consider the power law distribution of particles and get rid of  $f(p)$

$$\alpha_\nu = \frac{c^2}{8\pi h\nu^3} \int d^3p_2 [f(p_2^*) - f(p_2)] P(\nu, E_2)$$



$$N(E)dE = f(p)4\pi p^2 dp$$

$$\alpha_\nu = \frac{c^2}{8\pi h\nu^3} \int dE P(\nu, E) E^2 \left[ \frac{N(E - h\nu)}{(E - h\nu)^2} - \frac{N(E)}{E^2} \right] \quad (\text{replaced } E_2 \text{ by } E)$$



$$h\nu \ll E$$

$$\alpha_\nu = - \frac{c^2}{8\pi\nu^2} \int dE P(\nu, E) E^2 \frac{\partial}{\partial E} \left[ \frac{N(E)}{E^2} \right]$$

# Synchrotron self-absorption

$$\alpha_\nu = - \frac{c^2}{8\pi\nu^2} \int dE P(\nu, E) E^2 \frac{\partial}{\partial E} \left[ \frac{N(E)}{E^2} \right]$$



$$N(E) = KE^2 e^{-E/kT}$$

$$(\alpha_\nu)_{\text{thermal}} = \frac{c^2}{8\pi\nu^2 kT} \int N(E) P(\nu, E) dE = \frac{j_\nu c^2}{2\nu^2 kT}$$

This is Kirchhoff's law in Rayleigh-Jeans Regime (expected as  $h\nu \ll E$ )

# Synchrotron self-absorption

The last step is to consider the power law distribution of particles and get rid of  $f(p)$

$$N(E)dE = CE^{-p}dE$$

Do some algebra (Following R&L Sect 6.8)

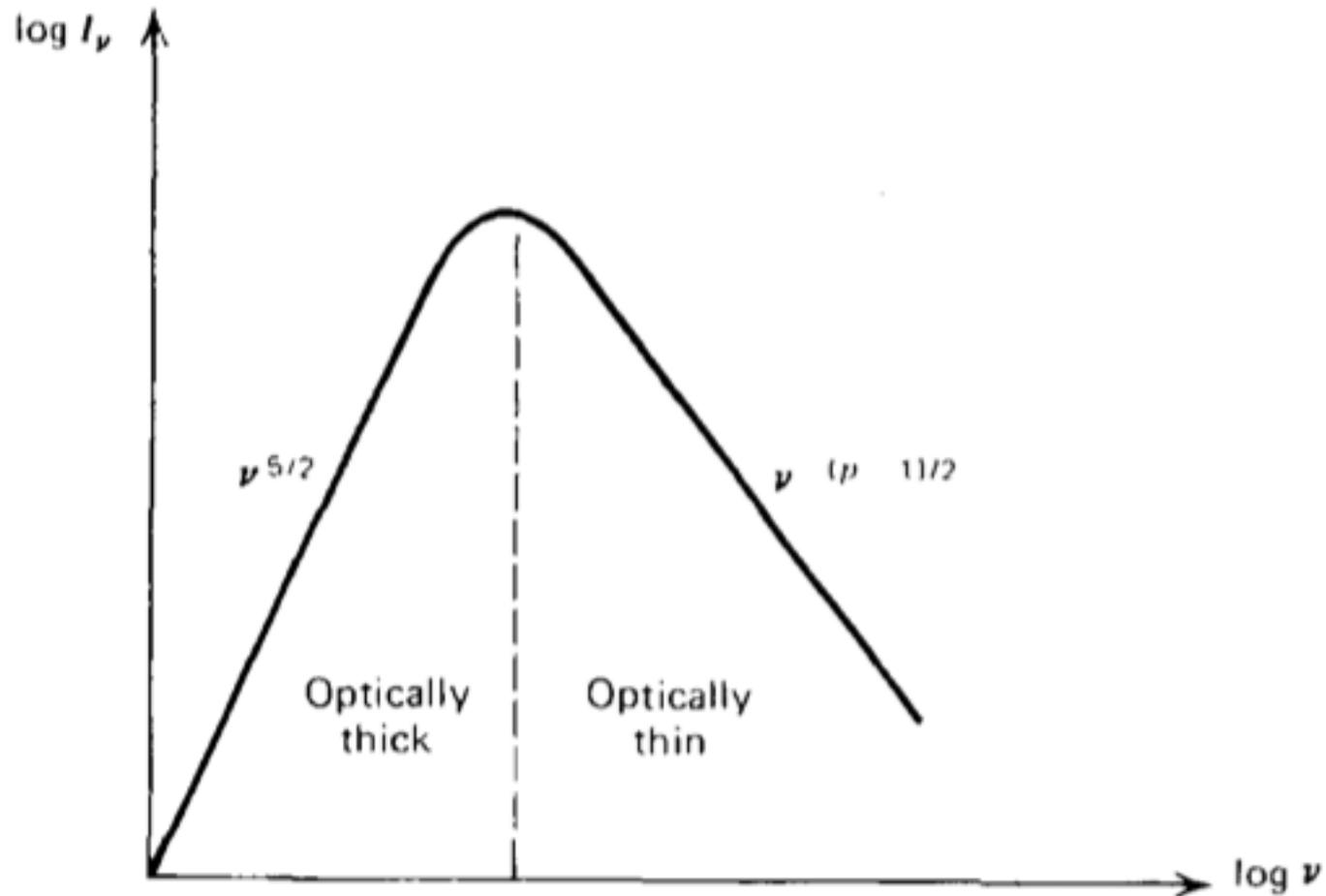
$$\alpha_\nu = \frac{\sqrt{3} q^3}{8\pi m} \left( \frac{3q}{2\pi m^3 c^5} \right)^{p/2} C (B \sin \alpha)^{(p+2)/2} \Gamma\left(\frac{3p+2}{12}\right) \Gamma\left(\frac{3p+22}{12}\right) \nu^{-(p+4)/2}$$

$$S_\nu = \frac{j_\nu}{\alpha_\nu} = \frac{P(\nu)}{4\pi\alpha_\nu} \propto \nu^{5/2}, \quad \text{Independent of } p$$

Note that the slope is not 2 as in the Rayleigh-Jeans regime, but it's 5/2.

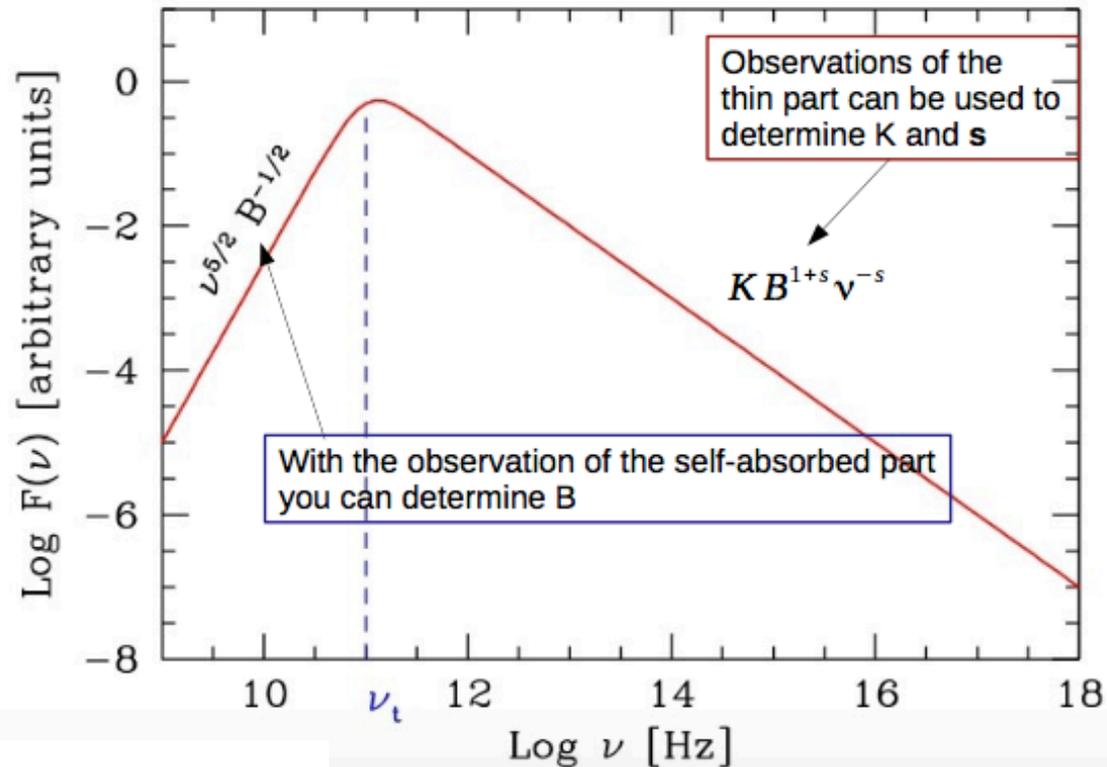
Here we do not have (and cannot have) a thermal distribution of particles  
emission is non thermal

# Synchrotron self-absorption



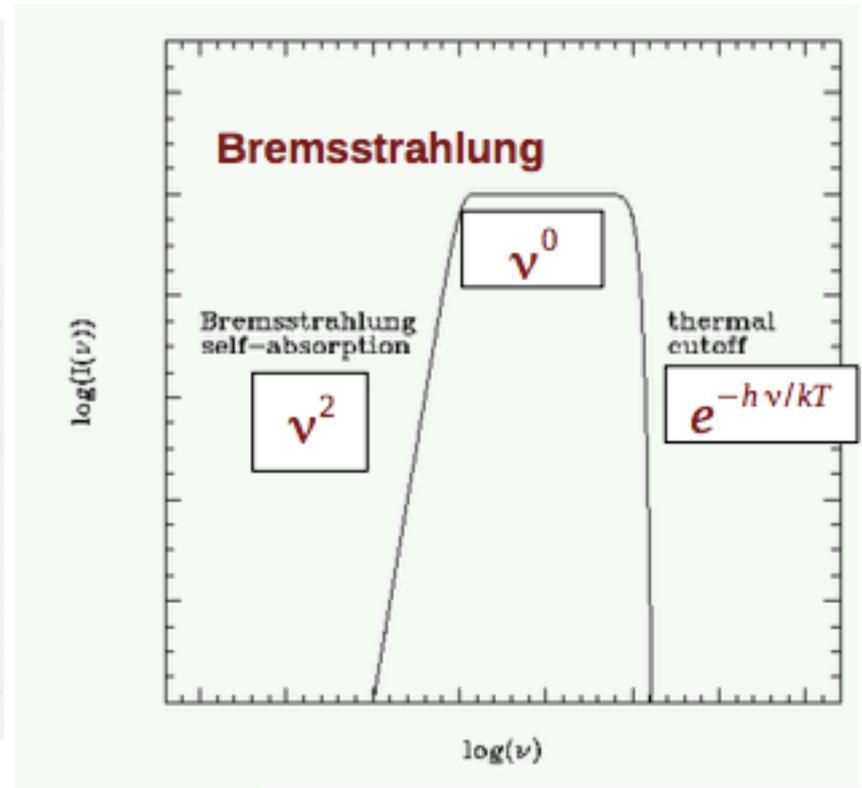
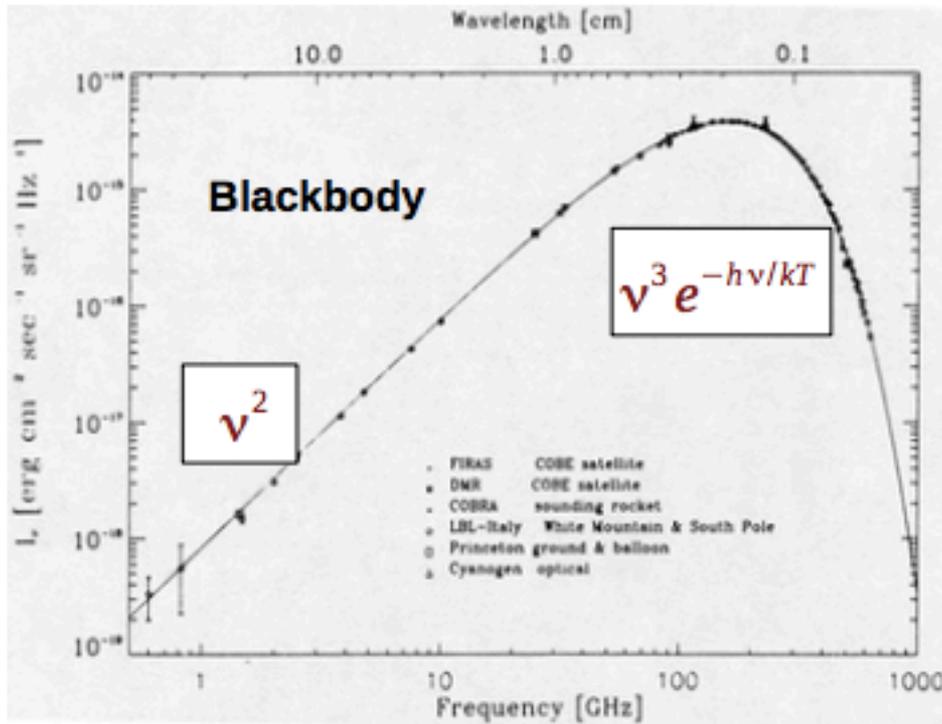
Synchrotron spectrum from power-law distribution of electrons

# Synchrotron self-absorption

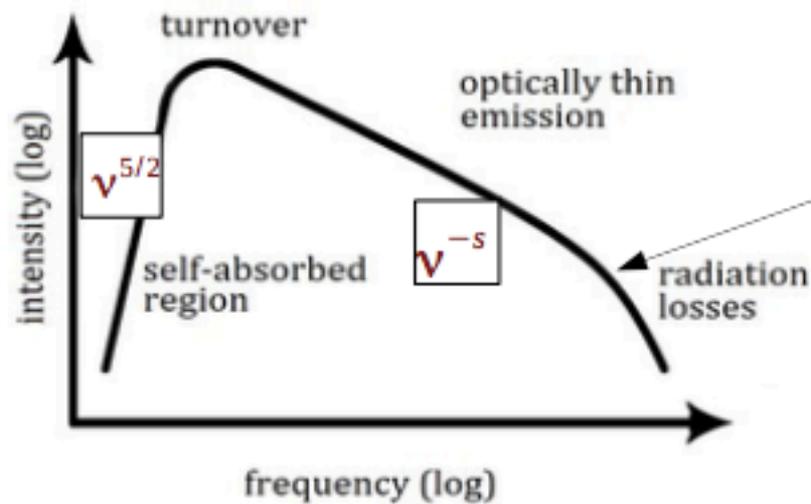


$$\nu_t \propto \left[ RK B^{(p+2)/2} \right]^{2/(p+4)}$$

Self absorption frequency: marks the transition from optically thin to thick



**Synchrotron**



What happens here?

# Synchrotron emission from Crab nebula



In the Crab nebula, spiraling electrons emitting optical photons have a lifetime of only  $\sim 100$  yr, and those emitting X-rays live only a few years. Such electrons could not have been accelerated in the 1054, supernova collapse that spawned the Crab nebula. Their energy source was a puzzle until the discovery of the Crab pulsar in 1968.

**The Crab Pulsar Powers the Nebula**

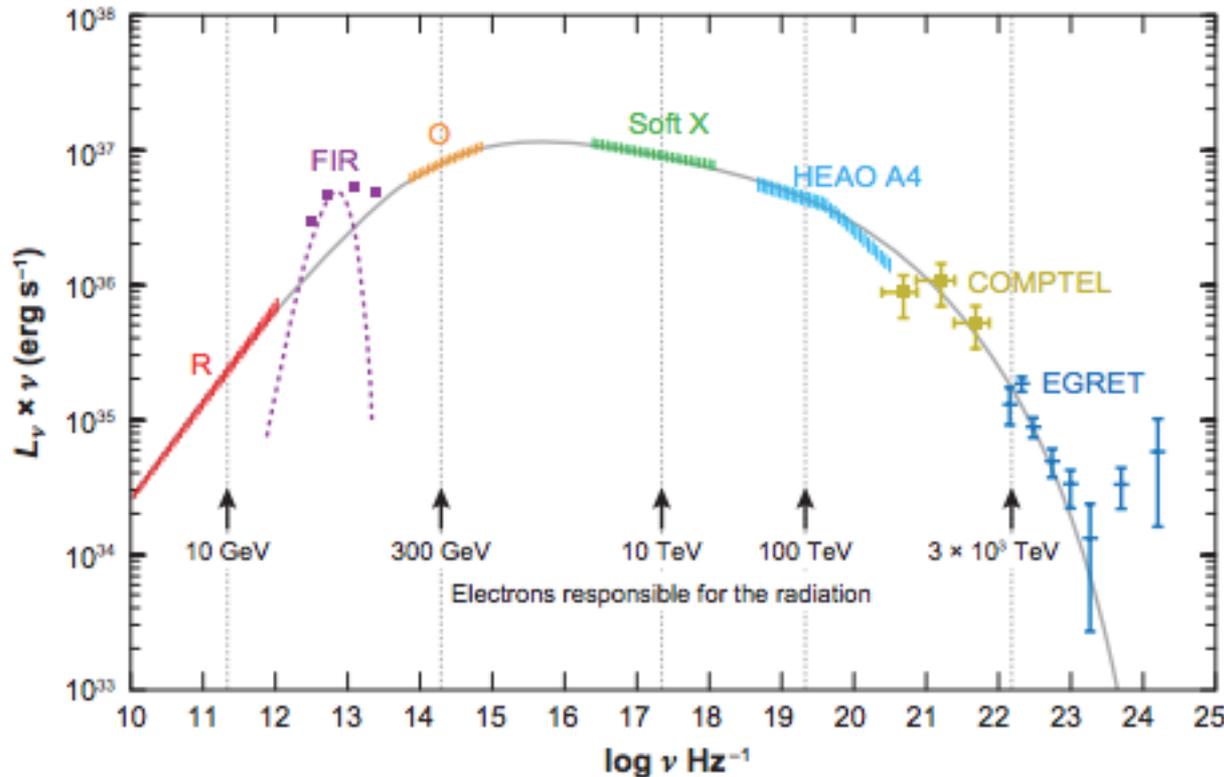
# Synchrotron emission from Crab nebula



Color composite of the Crab synchrotron nebula showing a Chandra X-ray image in blue, a visible light mosaic taken with HST in green, and a VLA radio image in red. The pulsar is seen as the bright blue point source at the center of the image.

Emission from high-energy electrons is brightest near the center of the nebula, close to where they are injected. Moving outward through the nebula, the spectrum becomes softer.

# Synchrotron emission from Crab nebula



The electron energies shown correspond to peak synchrotron emission assuming a magnetic field of  $300 \mu\text{G}$ . Most of the emission from the Crab is emitted between the optical and X-ray bands. The highest energy  $\gamma$ -rays are due to inverse Compton radiation.

Fig: The integrated spectrum of the Crab synchrotron nebula, from Atoyan & Aharonian (1996)

# Compton Scattering

Photon scattering by electrons - Overview		
$v \ll c$	<p><b>Low energy photons</b> <math>\hbar\omega \ll m_e c^2</math></p> <p><b>Thomson scattering</b> Classical treatment frequency unchanged</p>	<p><b>High energy photons</b> <math>\hbar\omega \geq m_e c^2</math></p> <p><b>Compton scattering</b> Quantum treatment incorporating photon momentum frequency decreases</p>
$v \sim c$	<p><math>\gamma\hbar\omega \ll m_e c^2</math></p> <p><b>Inverse Compton</b> Photons gain energy from relativistic electrons Approximate with classical treatment in electron rest frame Frequency increases</p>	<p><math>\gamma\hbar\omega \geq m_e c^2</math></p> <p><b>Inverse Compton</b> Quantum treatment in electron rest frame Photons gain energy from relativistic electrons</p>

# Thomson Scattering

(low-energy photons)

1. It occurs when the photon's energy is  $\ll$  electron rest mass
2. The electrons move non-relativistically:  $v \ll c$ .

The incoming and outgoing photon has the same energy and the electron does not change energy in the scattering process (elastic or coherent scattering).

# Compton Scattering

“In physics, Compton scattering or the Compton effect, is the decrease in energy (increase in wavelength) of an X-ray or gamma ray photon, when it interacts with matter.

Inverse Compton scattering also exists, where the photon gains energy (decreasing in wavelength) upon interaction with matter. The amount the wavelength increases by is called the Compton shift.”

Compton effect was observed by Arthur Holly Compton in 1923, for which he earned the 1927 Nobel Prize in Physics.

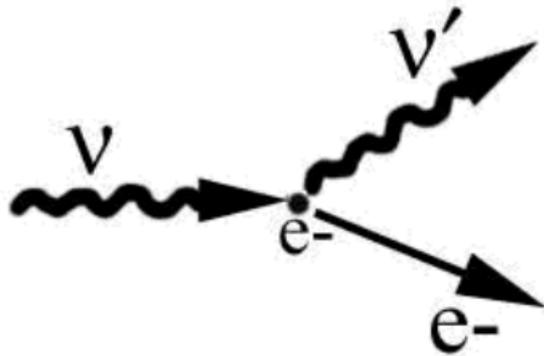
The effect is important because it demonstrates that light cannot be explained purely as a wave phenomenon.

Thomson scattering, the classical theory of charged particles scattered by an electromagnetic wave, cannot explain any shift in wavelength. Light must behave as if it consists of particles in order to explain the Compton scattering.



# Compton Scattering

Compton scattering

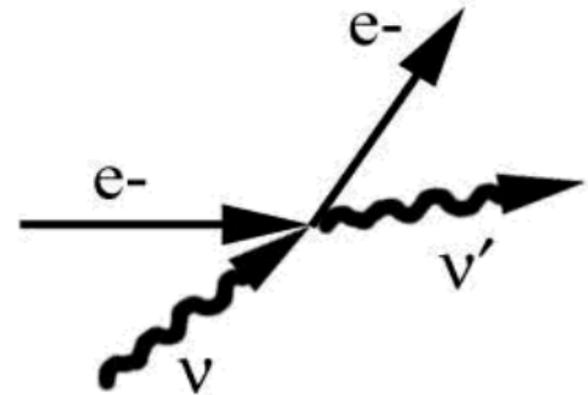


$$\nu' < \nu$$

Electron is initially at rest  
e- gains energy

Direct : Photon loses energy  
electron gains energy

Inverse Compton scattering



$$\nu' > \nu$$

High energy e- initially  
e- loses energy

Inverse : Photon gains energy  
electron loses energy

# Compton Scattering (notation)

- The prime symbol ' means that the quantity is calculated in the rest frame  $K'$  (i.e. the electron's rest frame in this case).
- No prime symbol means the quantity is calculated in the lab frame  $K$  (i.e. observer frame).
- The under-script 1 means that the quantity is calculated after the scattering has already occurred.
- No under-script means that the quantity is calculated before the scattering.

$\epsilon \rightarrow$  energy before the scattering in  $K$

$\epsilon_1 \rightarrow$  energy after the scattering in  $K$

$\epsilon' \rightarrow$  energy before the scattering in  $K'$

$\epsilon'_1 \rightarrow$  energy after the scattering in  $K'$

# Thomson Scattering

For low photon energies the scattering of radiation from free charges reduces to the classical Thomson scattering (Lecture 3,4).

$$\epsilon = \epsilon_1,$$

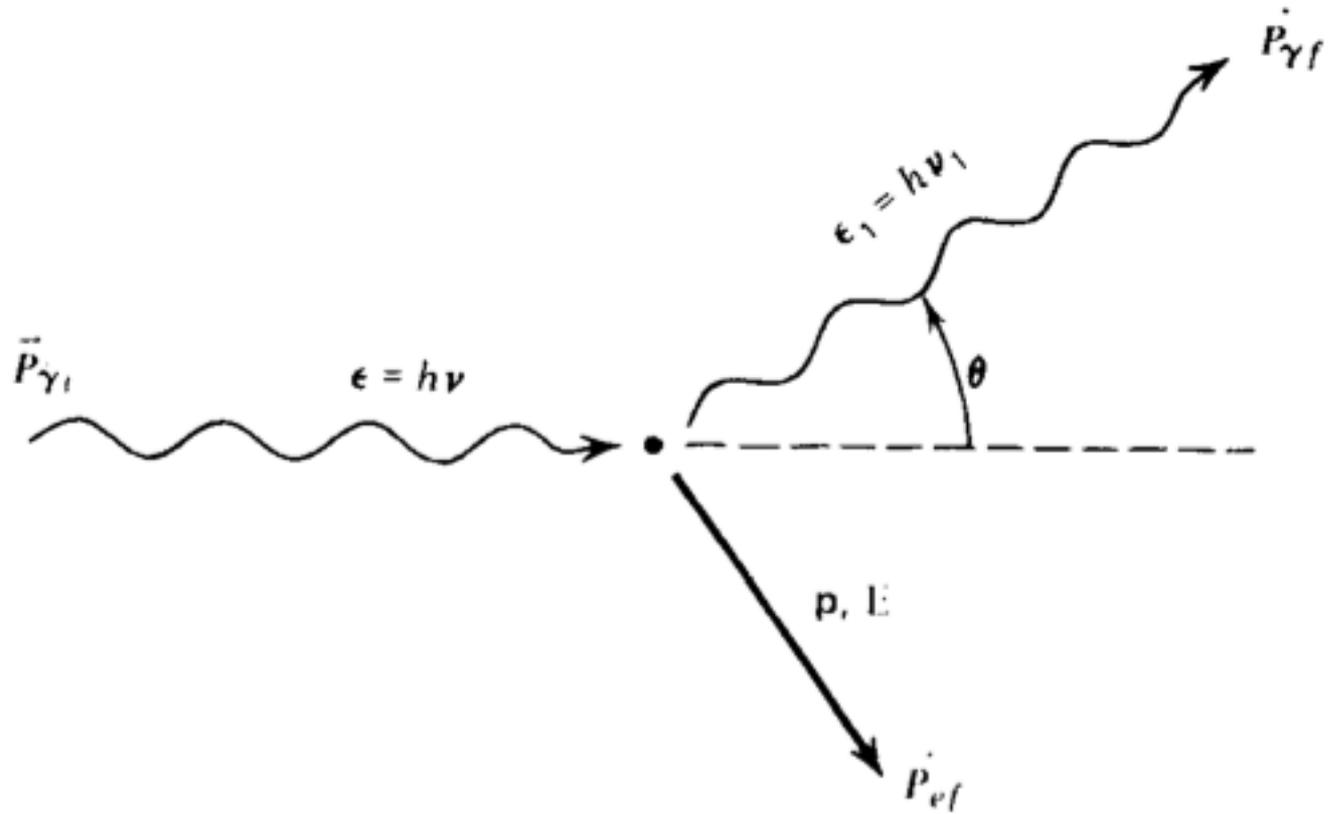


Equal energy for incident and scattered photons  
Elastic scattering

$$\frac{d\sigma_T}{d\Omega} = \frac{1}{2} r_0^2 (1 + \cos^2 \theta),$$

$$\sigma_T = \frac{8\pi}{3} r_0^2.$$

# Compton Scattering



# Compton Scattering

Let's start by looking at the momentum and energy of the photon and electrons.

In Thomson scattering the photon has no momentum (classical electrodynamics).

However, from quantum mechanics we do know that a photon has a momentum.

This means that scattering process cannot be purely elastic since the electron will recoil due to the momentum of the photon.

The photon has initial energy  $\epsilon$  and final energy  $\epsilon_1$ .

The photon has initial momentum  $\epsilon/c$  and final momentum  $\epsilon_1/c$ .

The electron has initial energy  $mc^2$  and final energy  $E/c$ .

The electron has initial momentum 0 and final momentum  $p$ .

# Compton Scattering

Using the conservation of energy and momentum it can be shown that the final and initial photon energies are related,

$$\epsilon_1 = \frac{\epsilon}{1 + \frac{\epsilon}{mc^2}(1 - \cos\theta)}.$$

In terms of wavelength this can be written as,

$$\lambda_1 - \lambda = \lambda_c(1 - \cos\theta)$$

Compton wavelength is defined as,

$$\lambda_c \equiv \frac{h}{mc} = 0.02426 \text{ \AA} \text{ for electrons}$$

# Compton Scattering

$$\lambda_1 - \lambda = \lambda_c (1 - \cos \theta)$$

The photon always loses energy, unless  $\theta = 0$ , and the scattering is closely elastic ( $h\nu \ll m c^2$ )

When the photons involved in the collision have large energies, the scattering becomes less efficient and quantum electrodynamics effects reduce the cross section.

The Thomson cross section becomes the Klein-Nishina cross section.

# Compton Scattering

When the wavelength of the incoming photon is smaller than the Compton wavelength then the Compton scattering is important.

The net effect is to decrease the energy of the photon. When the wavelength is larger than the Compton wavelength then elastic scattering (i.e., Thomson scattering) is a good approximation and the photon does not change wavelength (or energy).

Direct Compton

$$\lambda_1 - \lambda = \lambda_c (1 - \cos \theta)$$

Thompson Scattering

$$\lambda_1 - \lambda = 0$$

# Compton Scattering

Quantum effects appear in two ways

(1) Kinematics of the scattering process :

Since the photon has a momentum  $h\nu/c$  and energy  $h\nu$ , scattering will not be elastic because of recoil of charge

Need to consider energy momentum relations.

(2) Alteration of scattering cross section

# Compton Scattering

Differential scattering cross-section of unpolarized radiation is given by Klein-Nishina formula

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2} \frac{\epsilon_1^2}{\epsilon^2} \left( \frac{\epsilon}{\epsilon_1} + \frac{\epsilon_1}{\epsilon} - \sin^2\theta \right)$$

Note for  $\epsilon_1 = \epsilon$ , the scattering cross-section reduces to classical expression

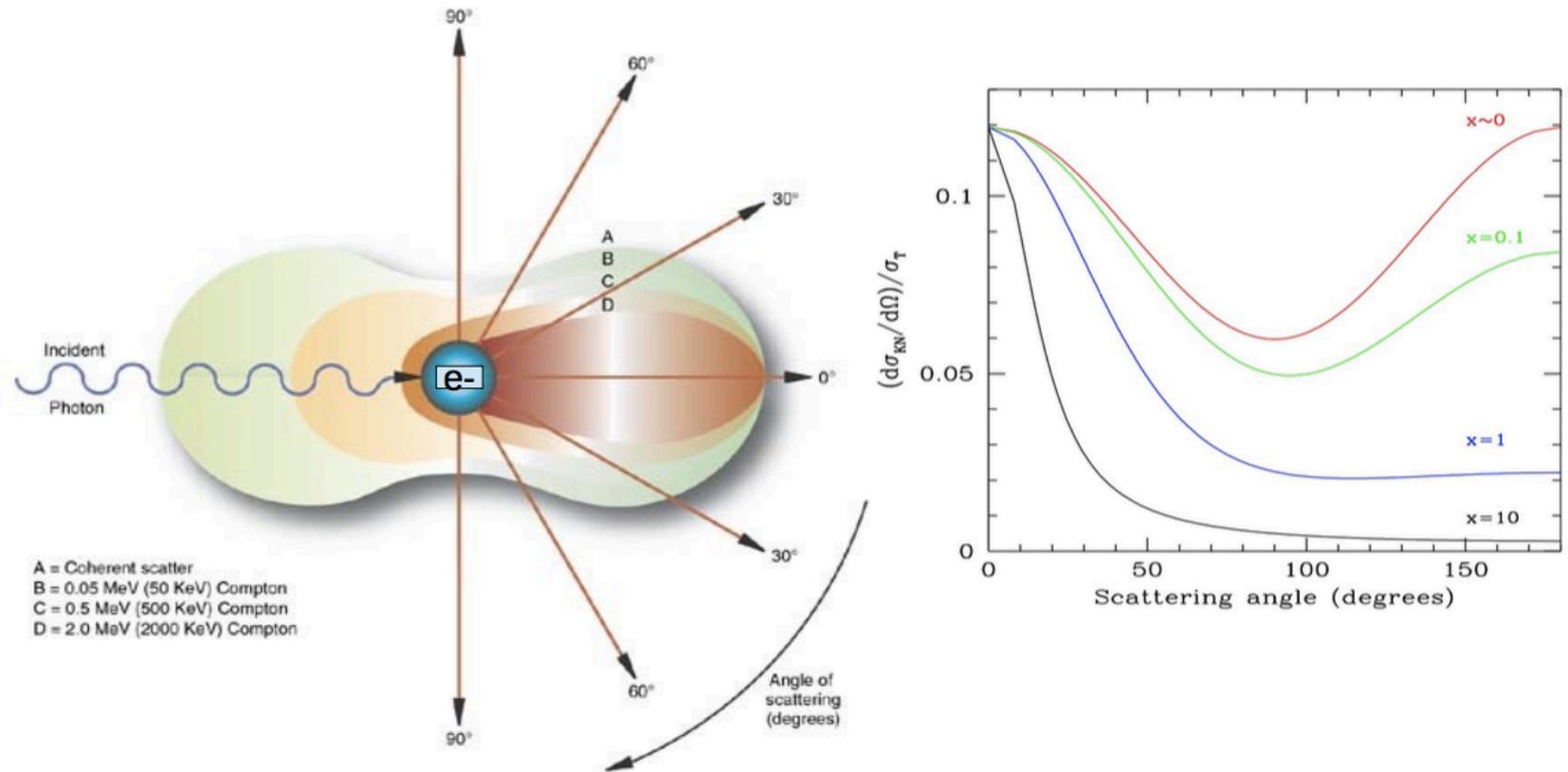
In the non relativistic regime we have (considering  $x = h\nu/mc^2$ )

$$\sigma \approx \sigma_T \left( 1 - 2x + \frac{26x^2}{5} + \dots \right) \quad x \ll 1$$

In extreme relativistic regime we have

$$\sigma = \frac{3}{8} \sigma_T x^{-1} \left( \ln 2x + \frac{1}{2} \right) \quad x \gg 1$$

# Compton Scattering



The green “peanut shape” pattern is the Thomson scattering ( $x \sim 0$ , coherent scattering)

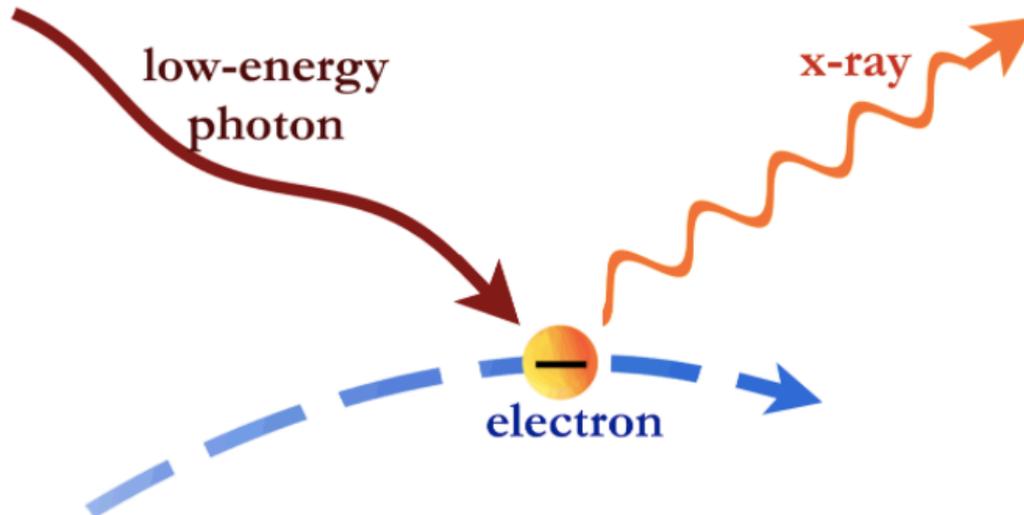
As the energy is increased the peanut shape disappears and the scattering becomes elongated in the “forward” direction.

# Inverse Compton Scattering

The direct Compton (or simply Compton) scattering is not a very common process in astrophysics, but its inverse process (inverse Compton) is more common.

Whenever moving electron has sufficient kinetic energy compared to the photon, net energy can be transferred from the electron to the photon : inverse Compton

How does the initial photon energy change after a collision with the relativistic electron?



# Inverse Compton Scattering (notation)

-- The prime symbol ' means that the quantity is calculated in the rest frame  $K'$  (i.e. the electron's rest frame in this case).

– No prime symbol means the quantity is calculated in the lab frame  $K$  (i.e. observer frame).

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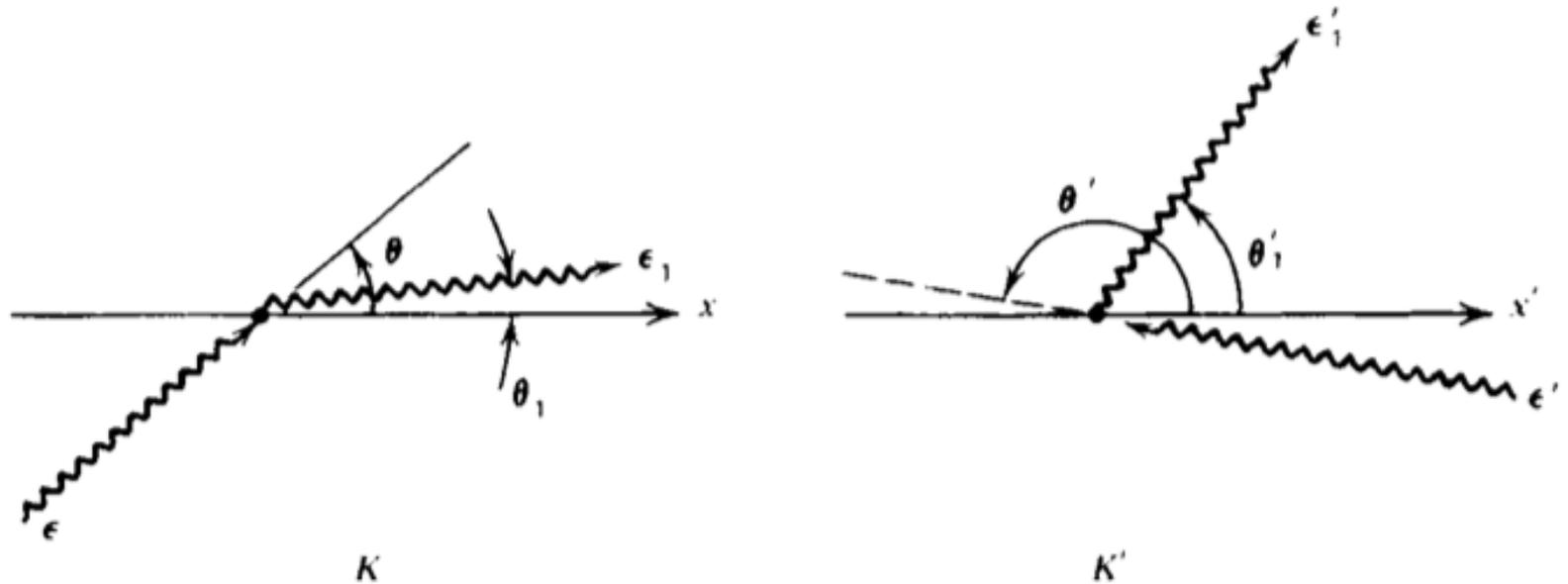
$\epsilon \rightarrow$  energy before the scattering in  $K$

$\epsilon_1 \rightarrow$  energy after the scattering in  $K$

$\epsilon' \rightarrow$  energy before the scattering in  $K'$

$\epsilon'_1 \rightarrow$  energy after the scattering in  $K'$

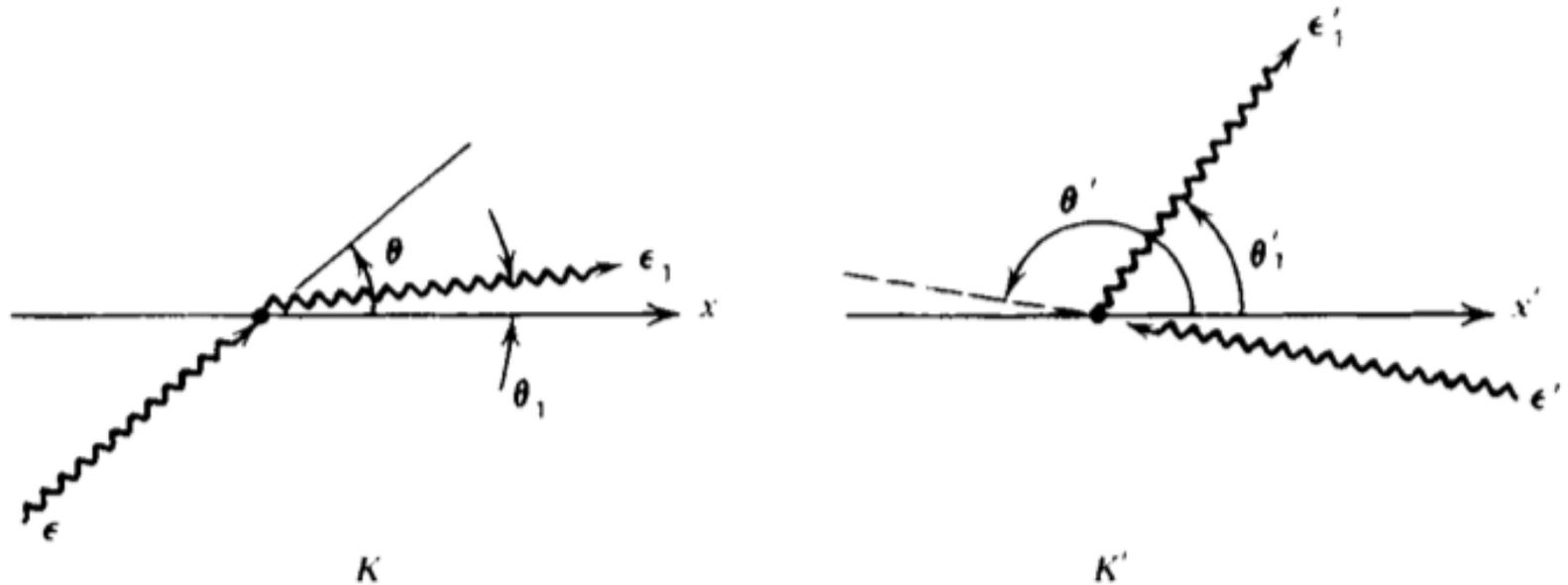
# Inverse Compton Scattering



Scattering geometries in the observer's frame K and the electron rest frame K'

- Step -1 : Photon and electron in lab frame to electron's rest frame
- Step-2 : Photon and electron interact in electron rest frame
- Step-3 : Back to the lab frame

# Inverse Compton Scattering



Scattering geometries in the observer's frame  $K$  and the electron rest frame  $K'$

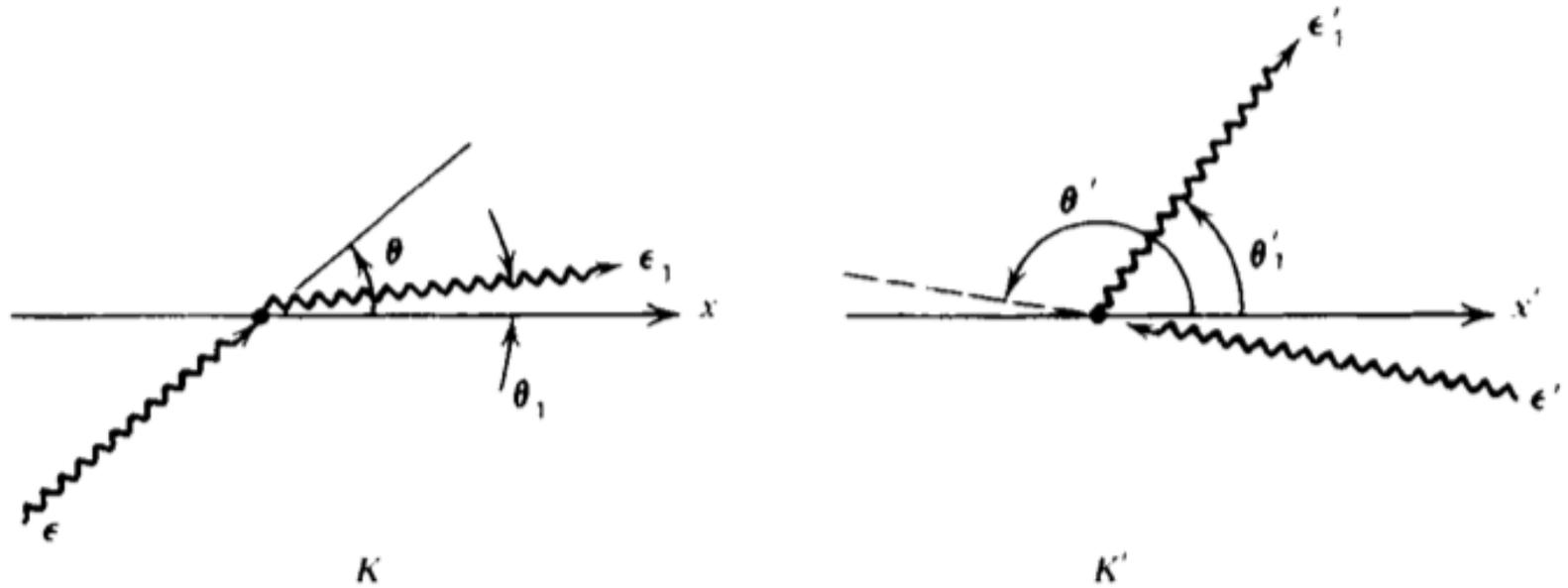
Step -1 : Photon and electron in lab frame and go to electron's rest frame

$\epsilon' = \epsilon\gamma(1 - \beta\cos\theta)$  ,  $\epsilon$  is energy of photon in  $K$

$\epsilon$  is energy of photon in  $K'$

Energy of photon is increased in electron's rest frame due to relativistic Doppler boost

# Inverse Compton Scattering



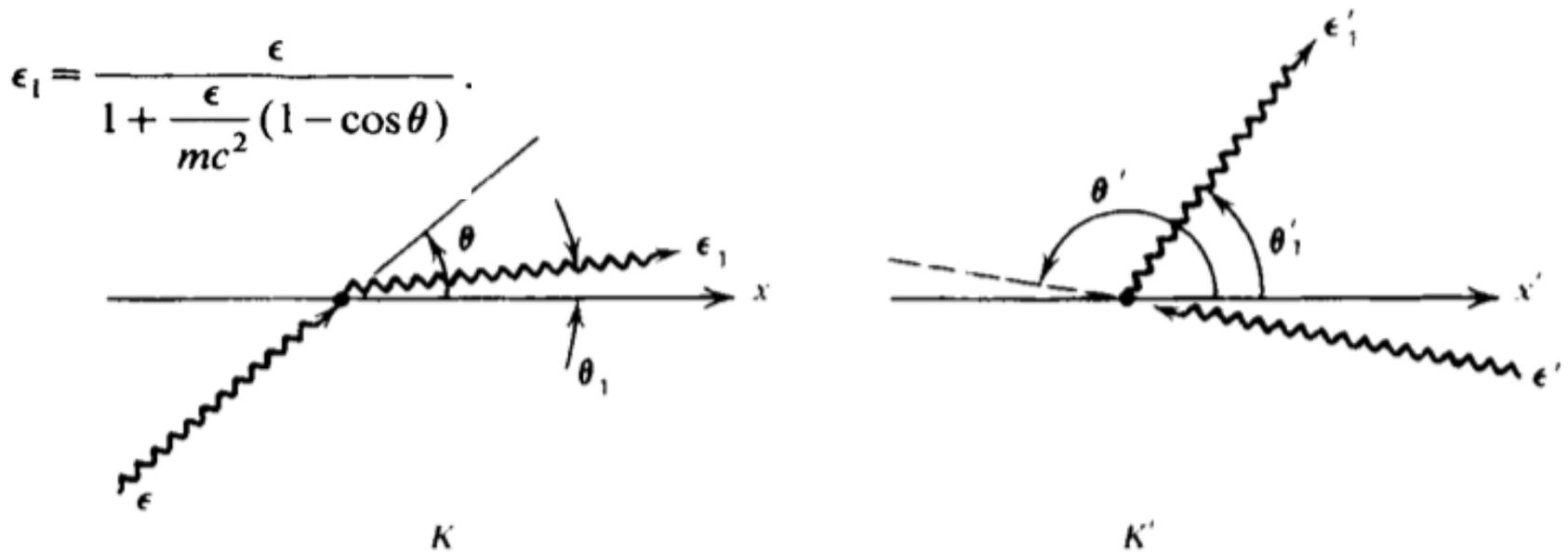
Scattering geometries in the observer's frame K and the electron rest frame K'

Step -2 : Photon and electron interact in electron's rest frame

$$\epsilon_1' = \frac{\epsilon'}{1 + \frac{\epsilon'(1 - \cos \Theta)}{mc^2}} \approx \epsilon' \left[ 1 - \frac{\epsilon'}{mc^2} (1 - \cos \Theta) \right]$$

We are now in electron's rest frame so we can use normal Compton formula

# Inverse Compton Scattering



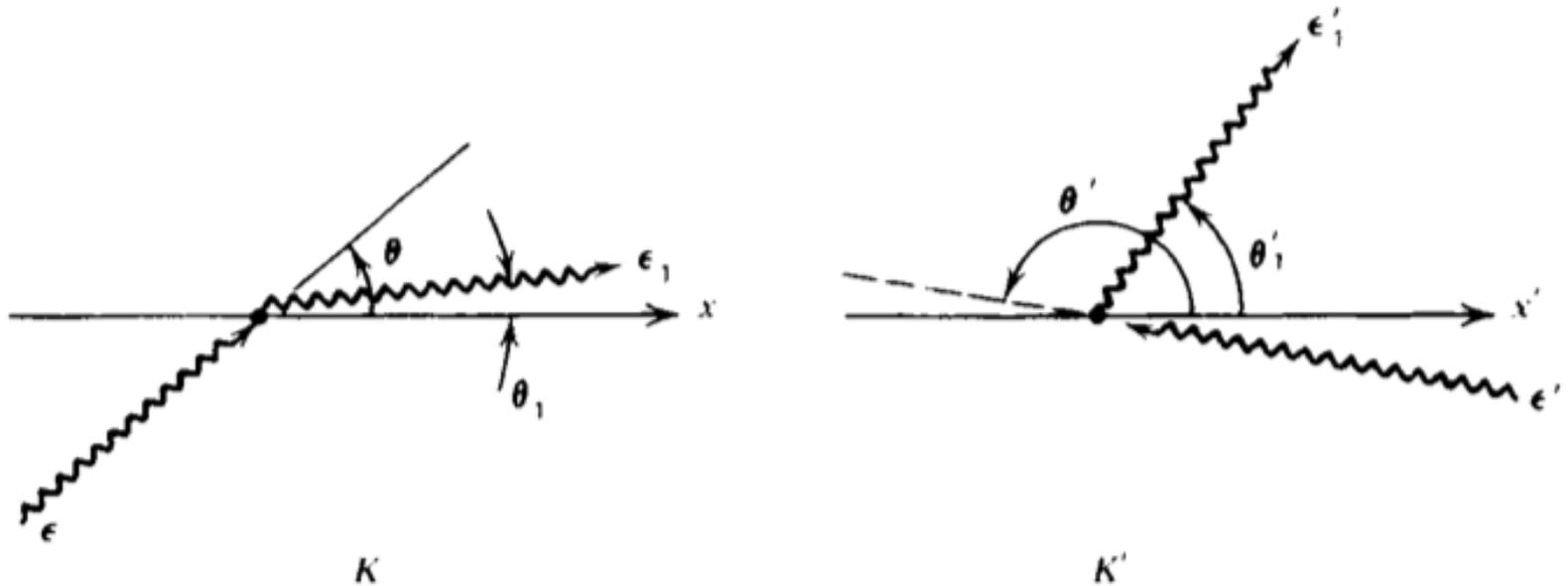
Scattering geometries in the observer's frame  $K$  and the electron rest frame  $K'$

Step -2 : Photon and electron interact in electron's rest frame

$$\epsilon_1' = \frac{\epsilon'}{1 + \frac{\epsilon'(1 - \cos\Theta)}{mc^2}} \approx \epsilon' \left[ 1 - \frac{\epsilon'}{mc^2}(1 - \cos\Theta) \right]$$

The photon energy has decreased in this process since some energy was given away to the electron

# Inverse Compton Scattering



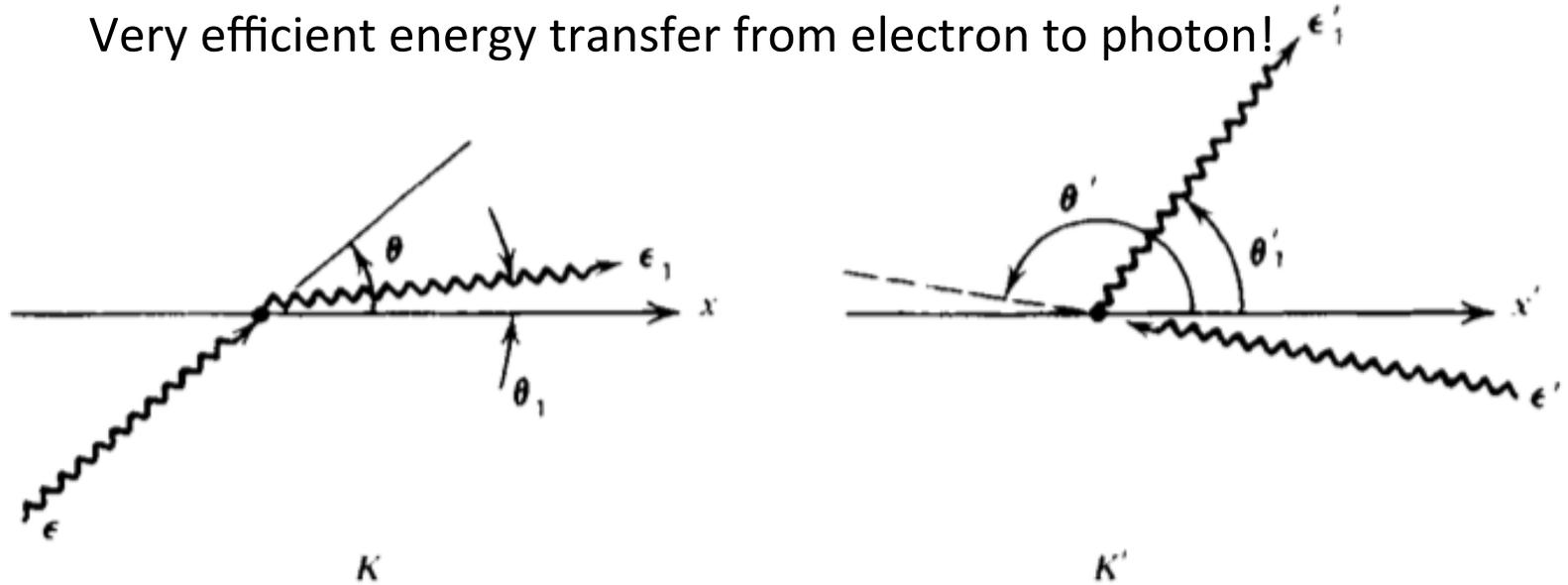
Scattering geometries in the observer's frame K and the electron rest frame K'

Step-3 : Back to the lab frame  $\epsilon_1 = \epsilon_1' \gamma (1 + \beta \cos \theta_1')$

We have a second Doppler boost because now we go back to the lab frame. The photon energy has increased again (second relativistic Doppler boost)

# Inverse Compton Scattering

Very efficient energy transfer from electron to photon!



Scattering geometries in the observer's frame  $K$  and the electron rest frame  $K'$

Summary:

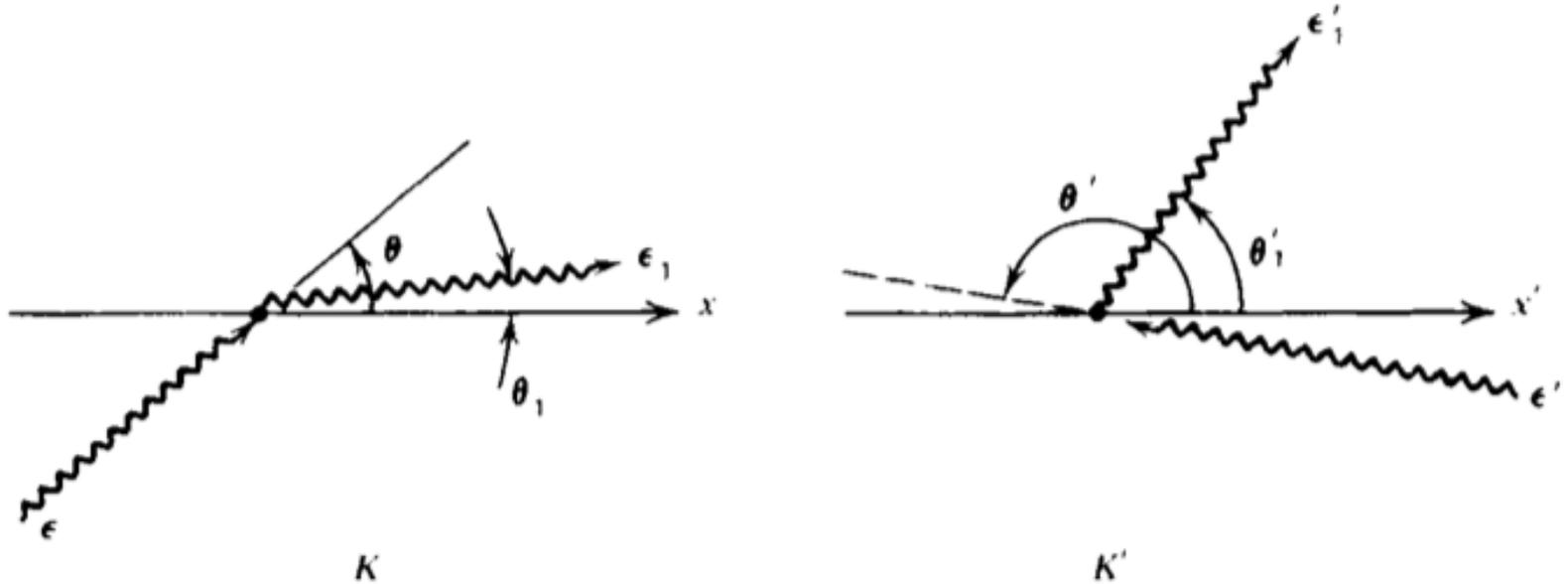
Step 1.  $K \rightarrow K'$  (1st Rel. Doppler boost)

Step 2. Compton Scattering (photon loses energy to the electron)

Step 3.  $K' \rightarrow K$  (2nd Rel. Doppler boost)

The photon gains an energy in step 1 and 3 by a factor  $\gamma$  (so in total it gains a factor  $\gamma^2$  because of the double Relativistic Doppler boost).

# Inverse Compton Scattering



Scattering geometries in the observer's frame  $K$  and the electron rest frame  $K'$

Step-3 : Back to the lab frame  $\epsilon_1 = \epsilon'_1 \gamma (1 + \beta \cos \theta'_1)$

Not all the quantities are calculated in lab frame  $K$

By transforming angle  $s$  from  $K'$  to  $K$

$$\epsilon_1 = \epsilon \frac{1 - \beta \cos \theta}{1 - \beta \cos \theta_1}$$

# Inverse Compton Scattering

## Minimum and Maximum energy

$$\epsilon_1 = \epsilon \frac{1 - \beta \cos \theta}{1 - \beta \cos \theta_1}$$

$\epsilon_1$  maximum when  $\theta = \pi$  and  $\theta_1 = 0$

$$\epsilon_1 = \epsilon \frac{1 + \beta}{1 - \beta}$$

Photon is scattered along velocity vector (head-on)

$\epsilon_1$  minimum when  $\theta = 0$  and  $\theta_1 = \pi$

$$\epsilon_1 = \epsilon \frac{1 - \beta}{1 + \beta}$$

Photon is scattered from behind velocity vector (tail-on)

# Inverse Compton Scattering

## Minimum and Maximum energy

$$\epsilon_1 = \epsilon \frac{1 - \beta \cos \theta}{1 - \beta \cos \theta_1}$$

$\epsilon_1$  maximum when  $\theta = \pi$  and  $\theta_1 = 0$       $\epsilon_1 = \epsilon \frac{1 + \beta}{1 - \beta}$      Photon is scattered along velocity vector (head-on)

$$\epsilon_1 = \epsilon \gamma^2 (1 + \beta)^2 \approx 4 \gamma^2 \epsilon$$



$$\nu' = 4 / 3 \gamma^2 \nu$$

In case it is possible to measure both the initial and final photon energy/frequency, then  $\gamma^2$  can be deduced

# Inverse Compton Scattering

The net power emitted by the electron in Compton scattering is,

$$P_{\text{compt}} = \frac{dE_{\text{rad}}}{dt} = \frac{4}{3} \sigma_T c \gamma^2 \beta^2 U_{\text{ph}}$$

Photon energy

Net power emitted in Synchrotron emission,

$$P_{\text{synch}} = \frac{4}{3} \sigma_T c \gamma^2 \beta^2 U_B$$

Very similar

$$\frac{P_{\text{synch}}}{P_{\text{compt}}} = \frac{U_B}{U_{\text{ph}}}$$

# Inverse Compton Scattering

$$\frac{P_{\text{synch}}}{P_{\text{compt}}} = \frac{U_B}{U_{\text{ph}}}$$

Radiation losses due to synchrotron emission and to Compton effect are in the Same ratio as the magnetic field density versus photon energy density

This is true for arbitrary value of electron velocities

# Inverse Compton Scattering

$$P_{\text{compt}} = \frac{dE_{\text{rad}}}{dt} = \frac{4}{3} \sigma_T c \gamma^2 \beta^2 U_{\text{ph}}.$$

$$P_{\text{synch}} = \frac{4}{3} \sigma_T c \gamma^2 \beta^2 U_B.$$

Why are these two powers so similar for very different physical mechanism operating?

The energy loss rate depends upon the electric field which accelerates the electron in its rest frame and it does not matter what the origin of that field is.

# Single particle spectrum

We want to know spectrum of the scattered radiation emerging after a collision with a single electron

Energy of the electron after scattering  $\epsilon_1 = \epsilon \frac{1 - \beta \cos \theta}{1 - \beta \cos \theta_1}$

Maximum Energy of the electron after scattering  $\epsilon_1 = \epsilon \frac{1 + \beta}{1 - \beta}$

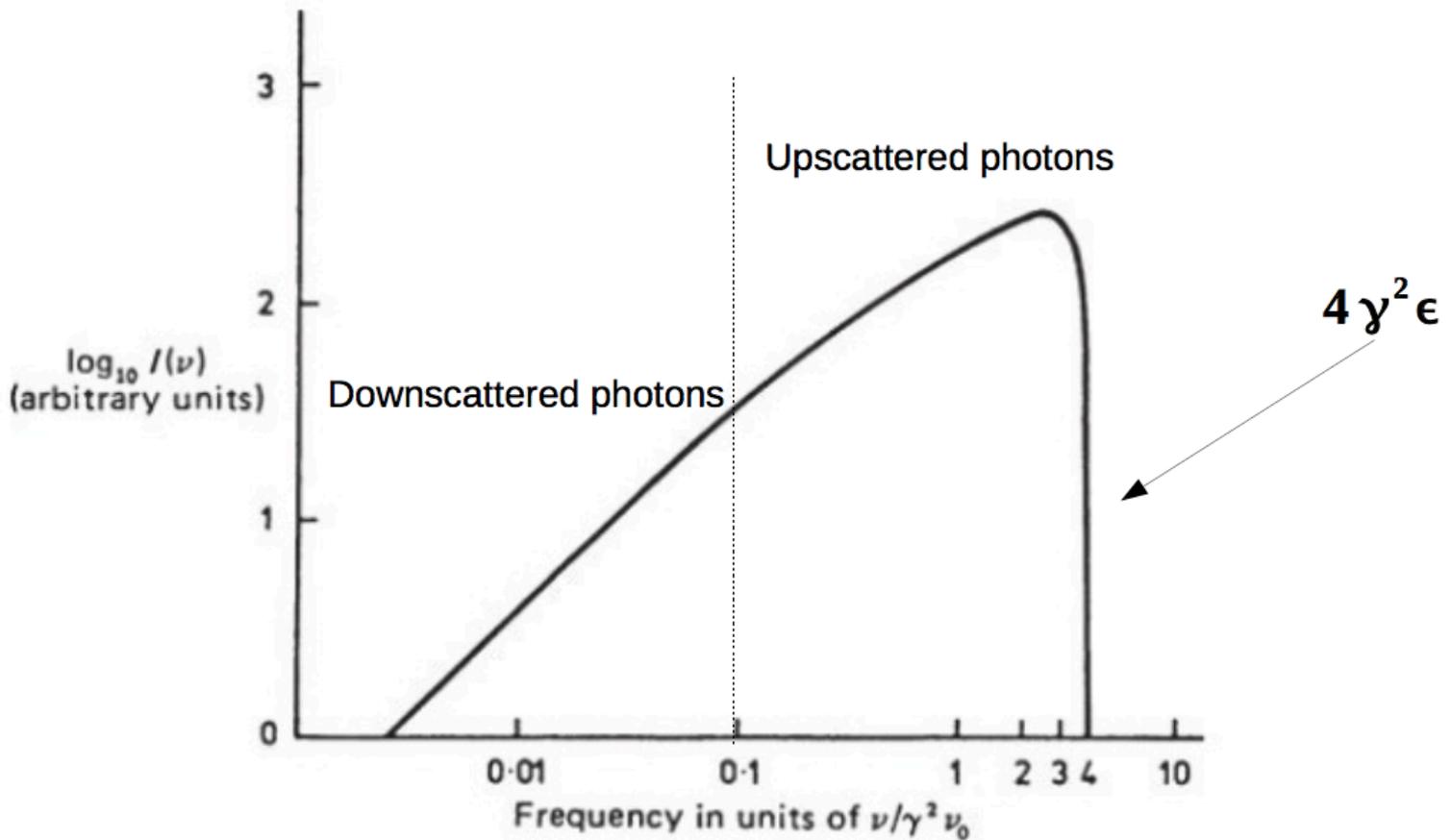
$$\epsilon_1 = \epsilon \gamma^2 (1 + \beta)^2 \approx 4 \gamma^2 \epsilon$$

So Inverse Compton spectra for single electron scattering should fall off at  $4\gamma^2 \epsilon$

Average photon energy for single electron scattering  $\sim (4/3) \gamma^2 \epsilon$

Multiple Scattering

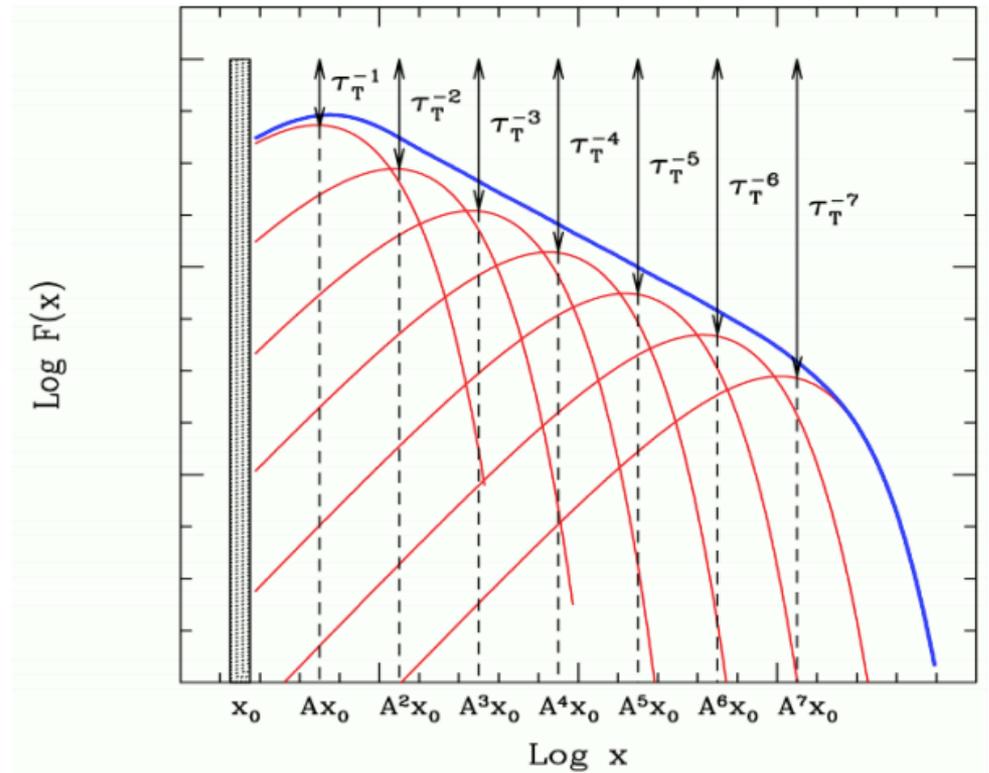
# Single particle spectrum



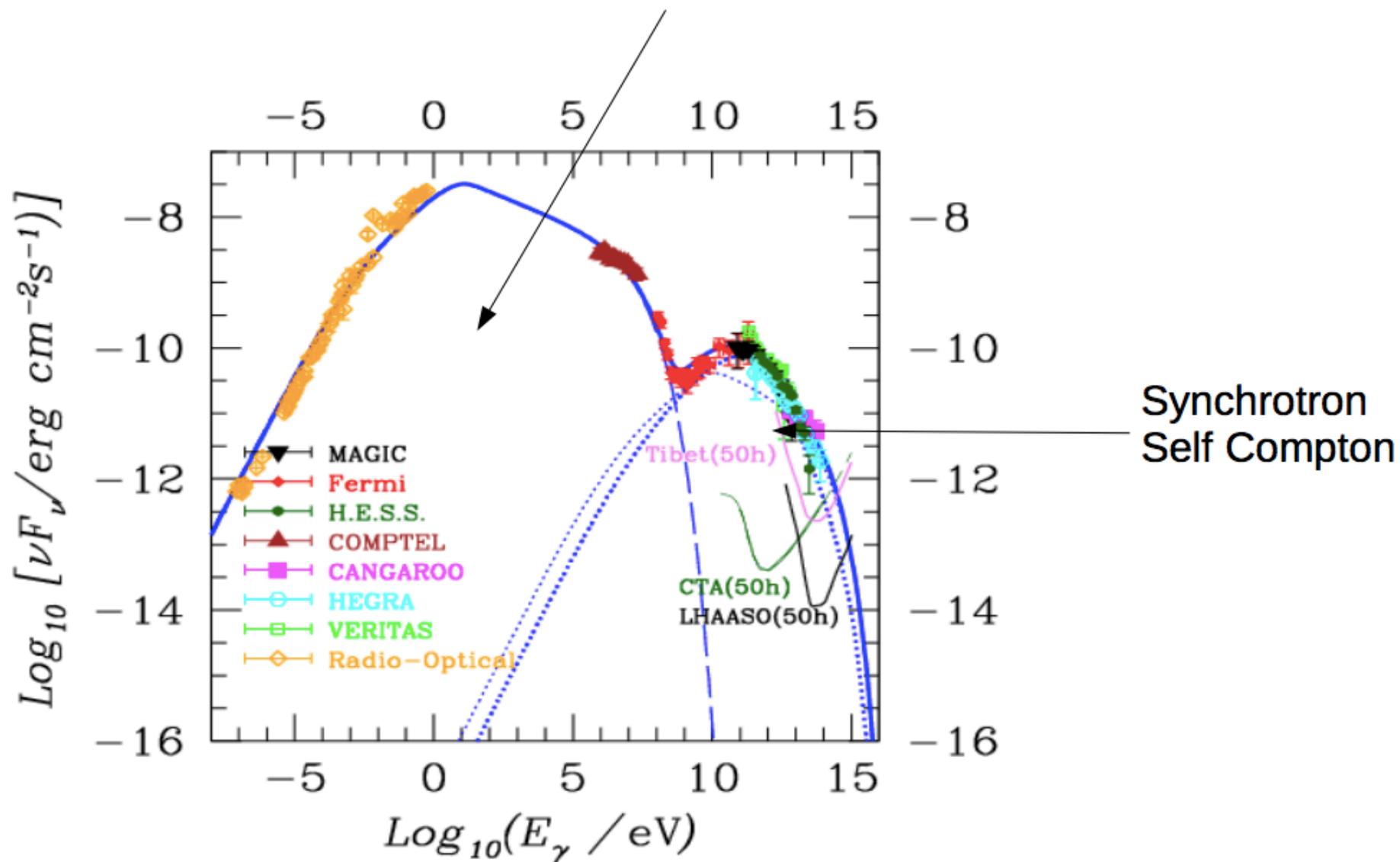
# Multi-scattering spectrum

Now, if the medium is of small optical depth the probability of a photon undergoing  $k$  scatterings before escaping the Comptonizing cloud is approximately:  
Prob.  $\approx \tau^k$

The intensity of the emerging Compton radiation will therefore decrease by a factor  $\tau$  as a function of energy (or frequency) of the radiation.



Synchrotron spectrum of the Crab Nebula.



# Synchrotron + inverse compton

The same relativistic electrons radiate via synchrotron and inverse Compton; their contributions add up.

The increased cooling rate implies that the electron radiative lifetime is consequently reduced.

# Astrophysical application

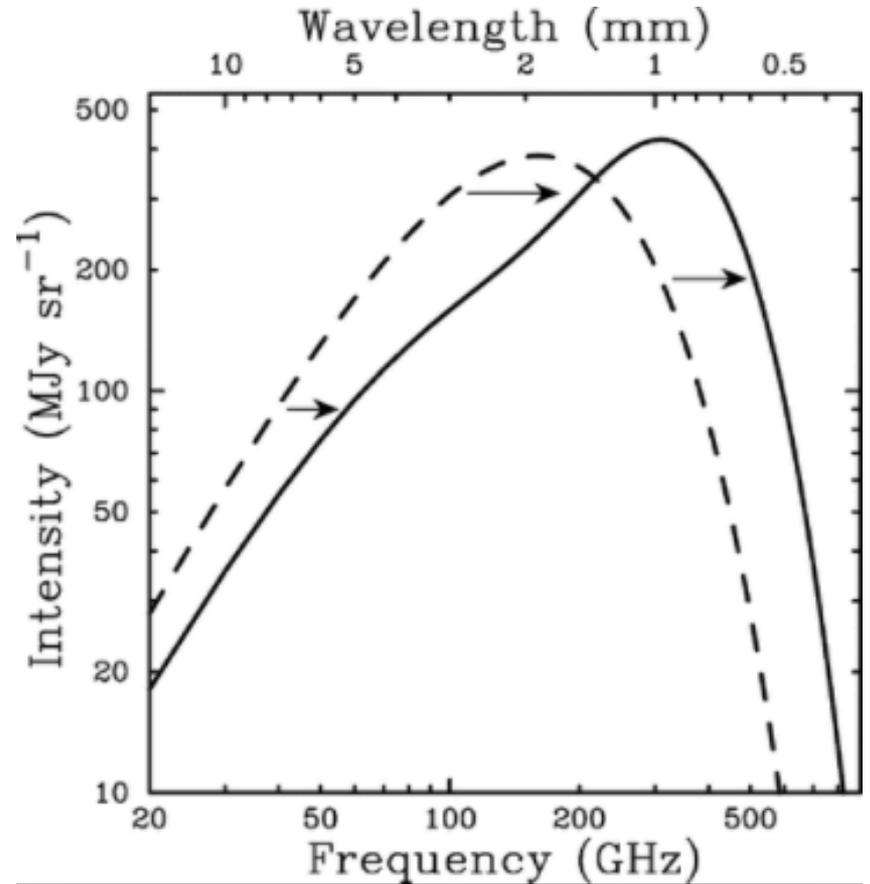
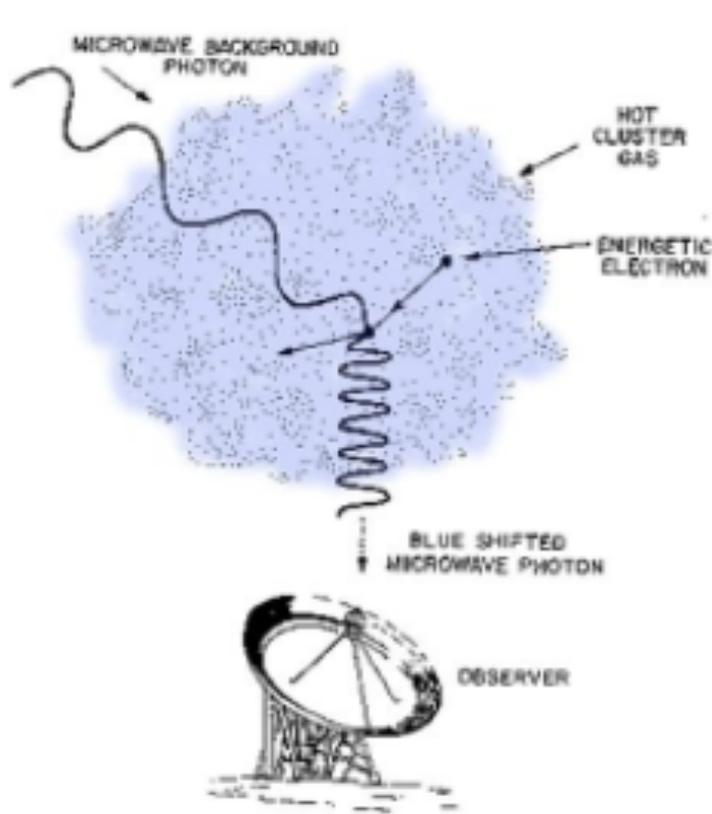
## Sunyaev-Zeldovich effect

Sunyaev–Zel'dovich effect (named after Rashid Sunyaev and Yakov B. Zel'dovich and often abbreviated as the SZ effect) is the distortion of the cosmic microwave background radiation (CMB) through inverse Compton scattering by high energy electrons in galaxy clusters, in which the low energy CMB photons receive an average energy boost during collision with the high energy cluster electrons.

SZ effect arises from inverse Compton scattering of cosmic microwave background (CMB) photons off energetic free electrons in the hot, ionized gas within galaxy clusters. This creates a small fluctuation in the CMB temperature along the line of sight towards a cluster.

Observed distortions of the cosmic microwave background spectrum are used to detect the density perturbations of the universe. Using the Sunyaev–Zel'dovich effect, dense clusters of galaxies have been observed.

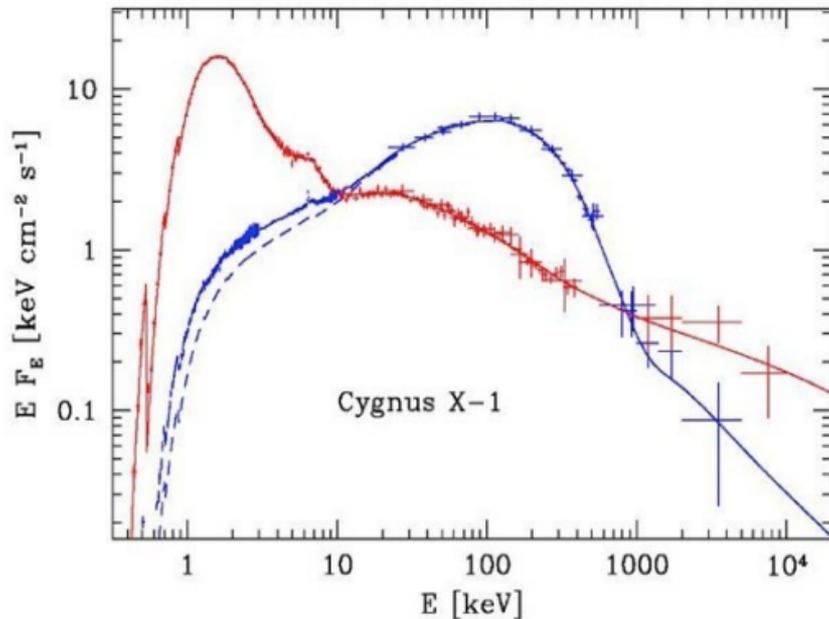
# Astrophysical application Sunyaev-Zeldovich effect



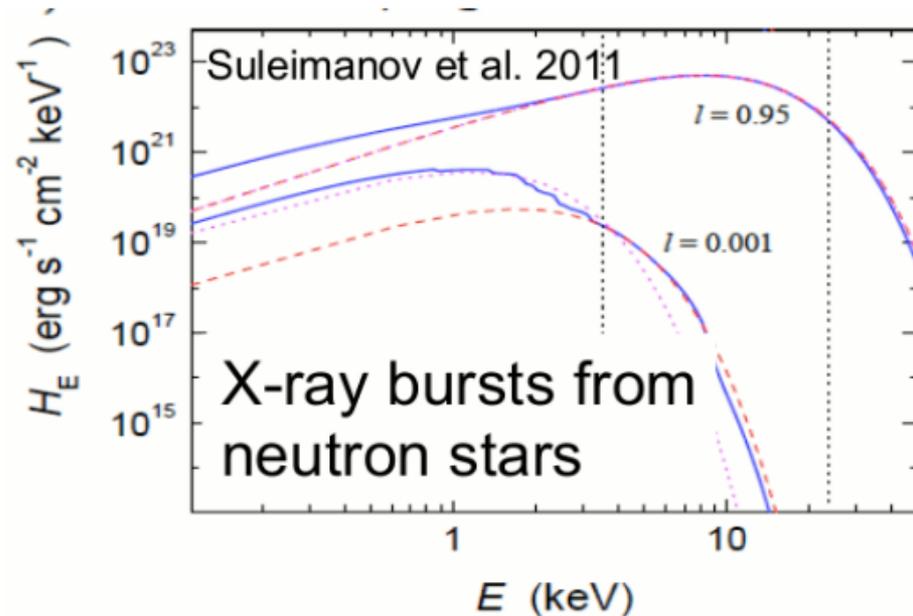
Refer: <https://astro.uni-bonn.de/~bertoldi/projects/sz/ringberg/img1.html>

# Other astrophysical applications

Accreting Black Holes & Neutron Stars



Thermonuclear bursts on Neutron stars



# End of Lecture 12

Next Lecture :19<sup>th</sup> September  
11:30-12:30