Electrodynamics and Radiative Processes I

Lecture 10 – Cyclotron to Synchrotron Radiation

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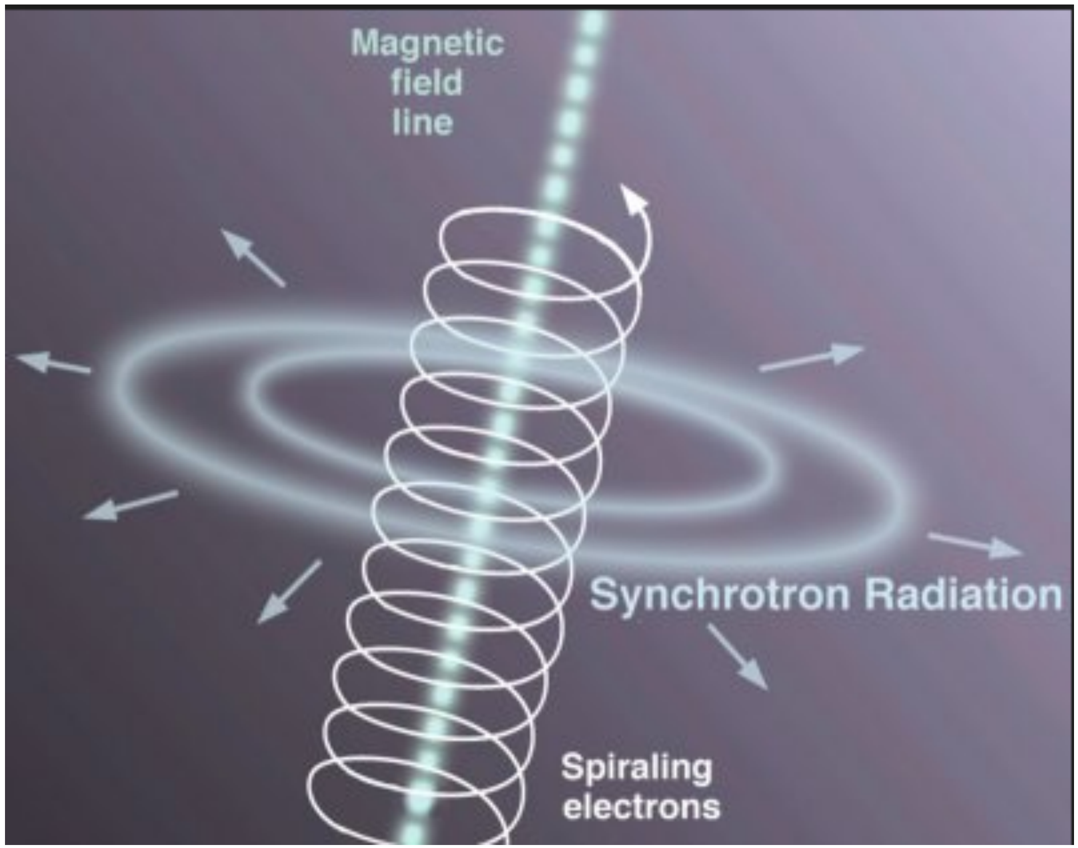
Synchrotron Radiation

Synchrotron Radiation is radiation from a charge moving relativistically that is accelerated by a magnetic field.



To understand synchrotron radiation let's first begin with the non-relativistic motion of a charge accelerated by a magnetic field : Cyclotron radiation

Cyclotron radiation

Particles accelerated by the magnetic field will radiate.

For nonrelativistic velocities nature of radiation is called **Cyclotron radiation**

Frequency of emission is frequency of gyration in the magnetic field.

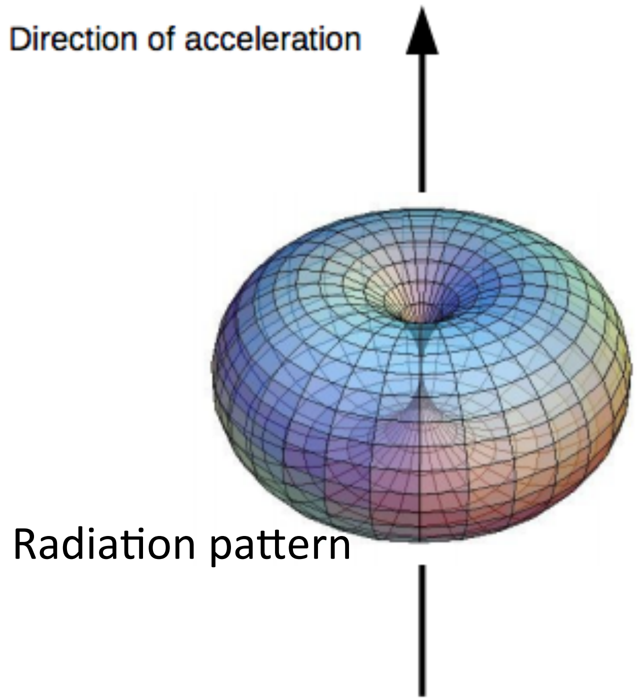
Cyclotron radiation

Accelerated charged particle will radiate according to the Larmor formula

$$P = \frac{2q^2\dot{u}^2}{3c^3}$$

It does not matter if the acceleration is given by electric field, gravity or magnetic field

Cyclotron radiation



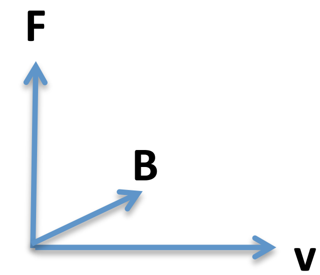
$$\frac{dW}{dt\,d\Omega} \approx \frac{q^2\dot{u}^2}{4\pi c^3}\sin^2\Theta$$

The radiation pattern is a torus with Sin^2 dependence of angle of radiation

Cyclotron radiation

Let us take a charge (say q) and put it in uniform magnetic field B

Force $F = q\,\mathbf{v} \times \mathbf{B} = q\,v\,B$ (If B is orthogonal to v)



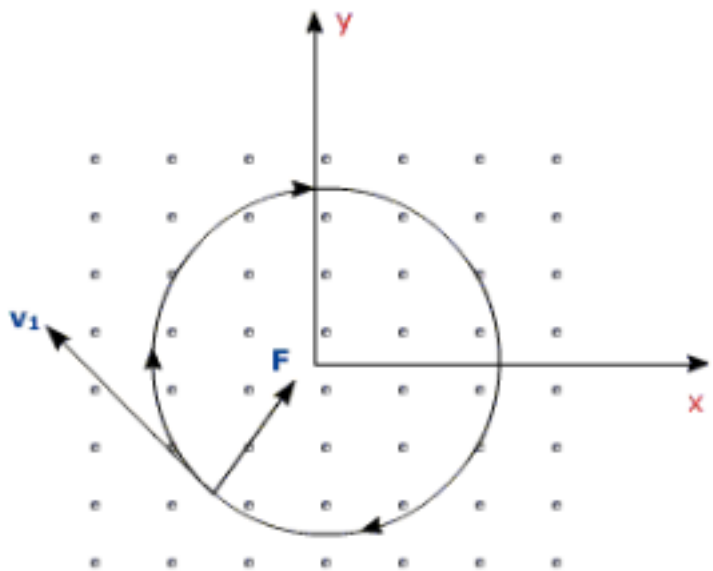
Force $F = q\,\mathbf{v} \times \mathbf{B} = q\,v\,B = mv^2/r_L =$ Centripetal force

Larmor Radius /Gyro Radius $r_L = mv/qB$

Force $F = mv^2/r_L = m\,\omega_L r_L$

Cyclotron frequency $\omega_L = qB/m$

Cyclotron radiation



From angular frequency we can find period of rotation of the charge

$$T=2\pi/\omega_L=2\pi m/qB$$

Period of the particle is not dependent on the size of orbit

Period of the particle is constant if B is constant

Cyclotron radiation

From angular frequency we can find period of rotation of the charge

$$T=2\pi/\omega_L=2\pi m/qB$$

The charge that is rotating will emit at a single specific frequency

$$\nu_L=\omega_L/2\pi=qB/2\pi m = 2.8 \text{ MHz per Gauss}$$

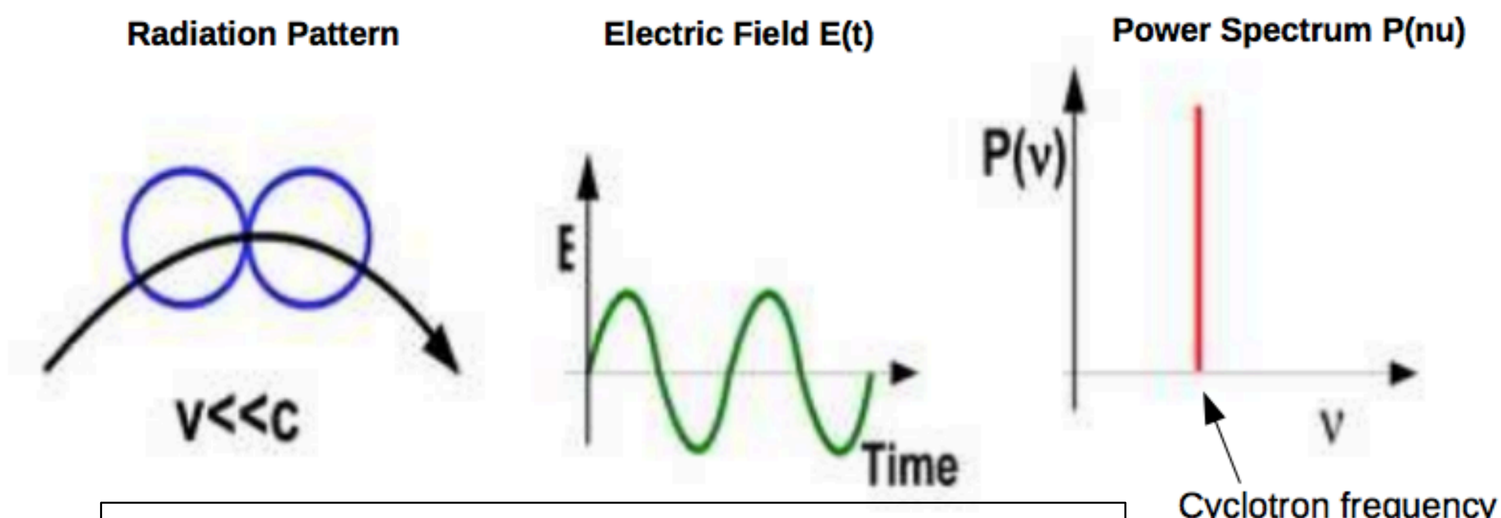
Frequency is independent of path radius and particle velocity

Cyclotron radiation

The emission appears at a single frequency

$$\nu_L = \omega_L / 2\pi = qB / 2\pi m = 2.8 \text{ MHz per Gauss}$$

The dipolar emission pattern is moving along the circle with constant velocity, the electric field measured will vary sinusoidally and the power spectrum will show a single frequency (the Larmor or cyclotron frequency).



Power spectra will peak at a single frequency

Cyclotron radiation
Kinetic energy

$(½)\ m\ v^2 = q\ V$

$v = \sqrt{2q\ V/m}$

protons need to be much more energetic than electrons to become relativistic

$$r_L = \frac{mv}{qB} \rightarrow r_L = \sqrt{\frac{2mV}{qB^2}}$$

Calculate Larmor radius for 1 Mev proton in 1 Tesla (~10^4 Gauss) field

Calculate Larmor radius for 1 Mev electron in 1 Tesla (~10^4 Gauss) field

Table 7.1 The properties of protons, carbon and iron nuclei having Lorentz factors $\gamma = 2$ and 100.

	Proton		Carbon nucleus		Iron nucleus	
Lorentz factor, γ	2	100	2	100	2	100
Velocity, v	$(\sqrt{3}/2)\,c$	$0.99995\,c$	$(\sqrt{3}/2)\,c$	$0.99995\,c$	$(\sqrt{3}/2)\,c$	$0.99995\,c$
Mass number, A	1	1	12	12	56	56
Atomic number, z	1	1	6	6	26	26
Rest mass energy, mc^2	1 GeV	1 GeV	12 GeV	12 GeV	56 GeV	56 GeV
Total energy, γmc^2	2 GeV	100 GeV	24 GeV	1200 GeV	112 GeV	5600 GeV
Kinetic energy, $(\gamma - 1)mc^2$	1 GeV	99 GeV	12 GeV	1188 GeV	56 GeV	5544 GeV
Kinetic energy per nucleon	1 GeV	99 GeV	1 GeV	99 GeV	1 GeV	99 GeV
Momentum, $pc = (\gamma m\|\boldsymbol{v}\|)c^{\dagger}$	$\sqrt{3}$ GeV	99.995 GeV	20.8 GeV	1199.9 GeV	96.99 GeV	5599.7 GeV
Rigidity, pc/ze	$\sqrt{3}$ GV	99.995 GV	$2\sqrt{3}$ GV	199.99 GV	3.73 GV	215.4 GV

† To obtain the dimensions of GeV, the momentum has been multiplied by c, the velocity of light.

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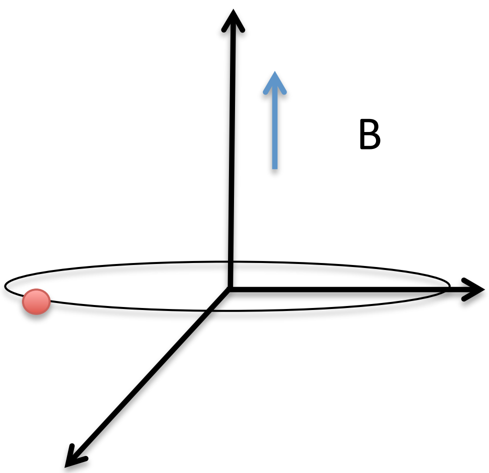
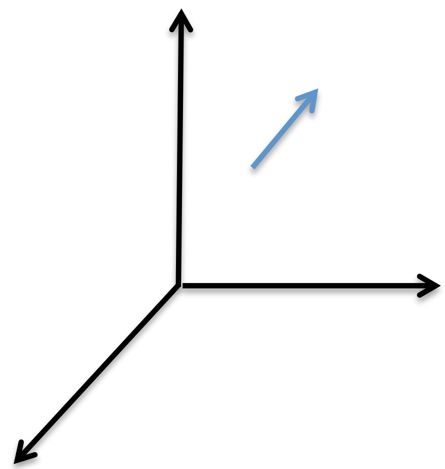
Cyclotron radiation
Polarization

B is perpendicular to LOS



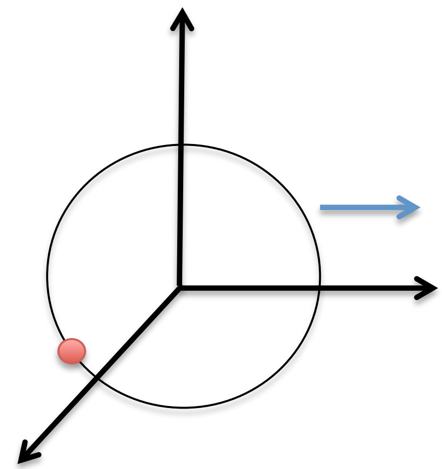
Linear Polarization

B is at an angle to LOS



Elliptical Polarization

B is parallel to LOS



Circular Polarization

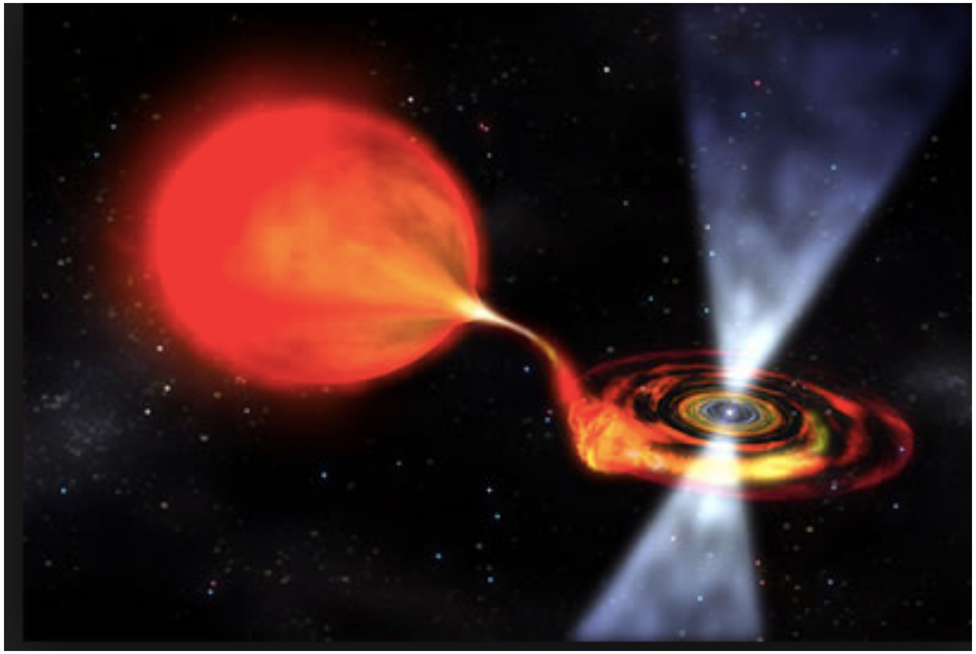
Polarization measurement to infer B strength and its orientation

Cyclotron radiation

Astrophysical application Cyclotron lines

Discovered ~ 40 years back

Cyclotron lines from the accreting x-ray pulsars

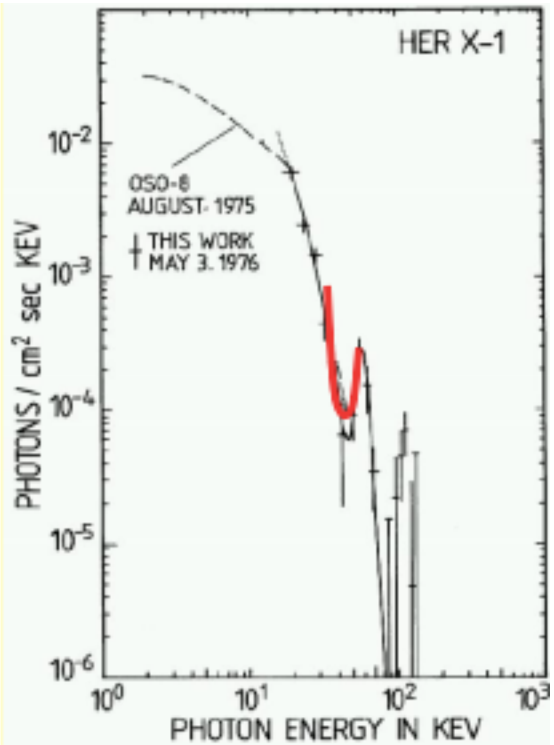


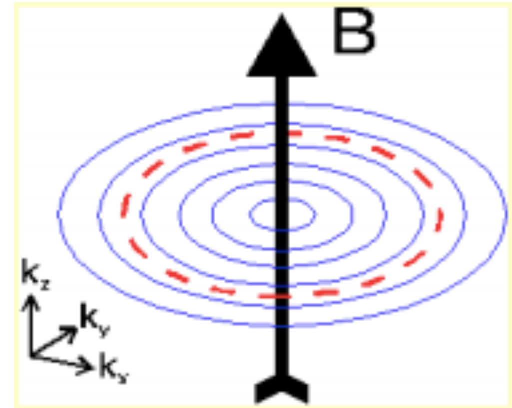
In 1977 J. Trumper identified a cyclotron emission line in the accreting pulsar Hercules X-1 :The X-ray spectrum shows an emission line at around 40 keV.

Trumper proposed : hot electrons around neutron star magnetic poles are rotating around a strong B field of ~5×10^{12} Gauss, giving rise to an emission line at ~40 keV.

Cyclotron lines

Astrophysical application



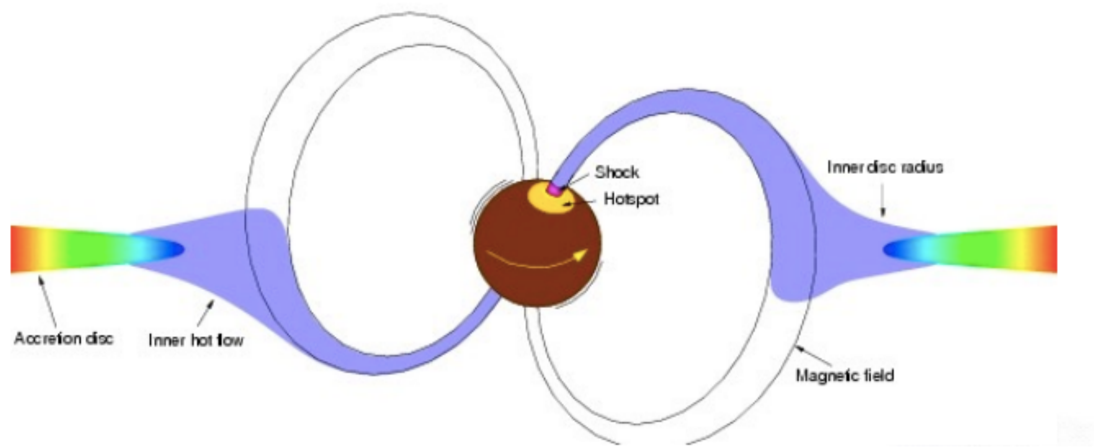


Directly probe the magnetic fields of the neutron stars

Probe geometry

Seen in more than 30 sources

Simulations + Observations



Geometry of Her X1

Cyclotron radiation

Astrophysical application

Discovered ~ 40 years back

Estimate the magnetic field if you get a cyclotron absorption feature at 34 KeV?

Cyclotron radiation

Astrophysical application

Discovered ~ 40 years back

A substantial fraction of the known neutron stars reside in X-ray binaries, providing an ideal site to study these objects.

Neutron star binary systems/ accretion powered pulsars (ACPs), accrete matter from the companion and emit pulsed radiation at X-ray wavelengths.

Accretion powered X-ray pulsars are some of the most powerful sources of X-ray radiation in our Galaxy.
Luminosity within $10^{33} - 10^{35}$ erg s^{-1} during quiescence
Luminosity rise up to 10^{38} erg s^{-1} during active state
Strong magnetic fields up to 10^{11} - 10^{13} G

Cyclotron lines provided the first direct measurement of the magnetic field strength of a neutron star

Ref: Maitra et al 2017

Cyclotron radiation

Astrophysical application

Discovered ~ 40 years back

Cyclotron lines are usually detected as absorption lines in the continuum spectrum, and are modelled phenomenologically with Gaussian or pseudo-Lorentzian profiles

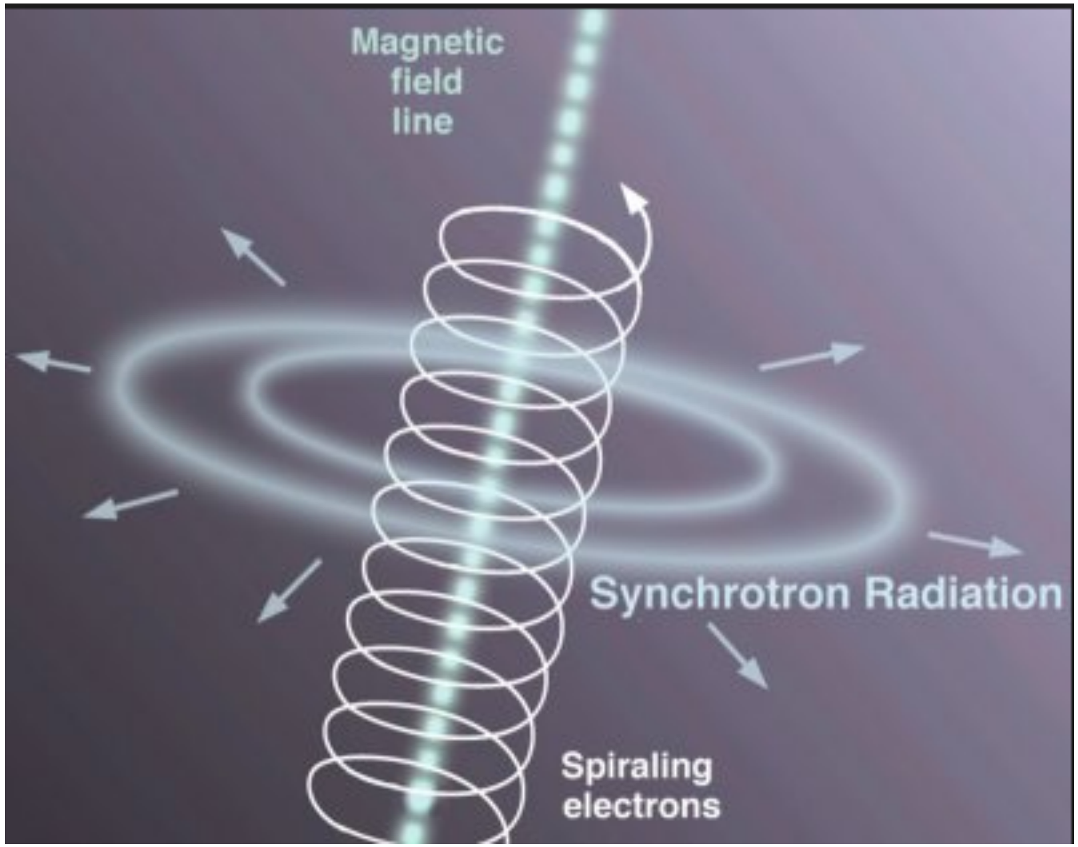
More than 30 such sources known

Changes of line parameters with luminosity provides probe of geometry

Ongoing missions NuSTAR and ASTROSAT are probing Cyclotron lines.

"Modelling the timing and spectral results jointly with the latest physical models can provide a comprehensive picture on the physics of these accreting binary pulsar systems" (Maitra et al. 2017)

Synchrotron Radiation is radiation emerging from a charge moving relativistically that is accelerated by a magnetic field.



Emission by ultra-relativistic electrons spiraling around magnetic field lines

Relativistic effects: from Cyclotron to Synchrotron Radiation

Assumption v<<c (non relativistic particles) **for Cyclotron**

Now we describe what happens to the radiation of a charge accelerated in a B field when the speeds approach c **for Synchrotron**

Review Relativistic effects discussed in Lecture 5

Lorentz transformations of time: $\Delta t = \Delta t' \gamma$

Lorentz transformations of Frequency: $\nu = \nu' / \gamma$

Relativistic effects: from Cyclotron to Synchrotron Radiation

Cyclotron → **Synchrotron**

Larmor Frequency → Frequency of Gyration

$\nu_L = \omega_L / 2\pi = qB/2\pi m$ → $\nu_B = \omega_B / 2\pi = qB/2\pi m\gamma$

Larmor radius → Radius of Gyration

$$r_L = \frac{mv}{qB} \rightarrow r_L = \sqrt{\frac{2mV}{qB^2}}$$

$$r_L = \sqrt{\frac{(\gamma+1)mV}{qB^2}} = \frac{\gamma m v}{qB}$$

Period of rotation → Period of rotation

$T = 2\pi/\omega_L = 2\pi m/qB$ → $T = 2\pi/\omega_B = 2\pi m\gamma/qB$

The period depend on particle velocity (Lorentz factor gamma) and as the velocity approaches c, the period increases.

Synchrotron Radiation

"Synchrotron" in synchrotron machines: the strength of the B field is not kept constant, but it is increased with time so that as gamma increases the frequency and the radius of gyration are constant.

Very famous Synchrotron machine : LHC (Large Hadron Collider)

↓

Synchrotron machine used to generate relativistic protons up to 7 TeV in energy (per beam).

Synchrotron Radiation
In Astrophysics

Magnetic fields and relativistic particles are prerequisite for synchrotron radiation in astrophysics.

So synchrotron emission is seen in a wide variety of environments.

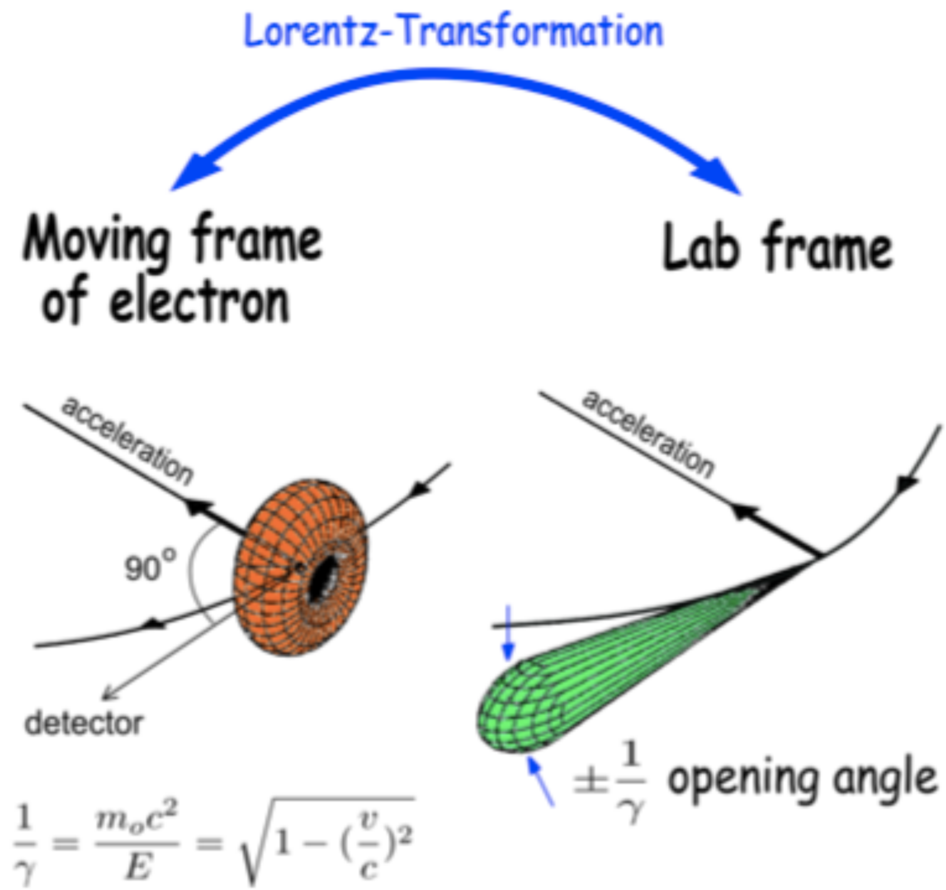
Typical magnetic field strengths

Location	Magnetic filed (Gauss)
Interstellar medium	10^{-6}
Stellar atmosphere	1
Black hole	10^{4}
White dwarf	10^{2}
Neutron star	10^{12}
Earth	0.3

Synchrotron Radiation
Emission pattern

A relativistic electron moving around a B field.

Cyclotron to Synchrotron:
- start with the radiation pattern in the electron rest frame (where we know the radiation pattern)
- then we do a Lorentz transformation from the rest frame to the lab frame.



End of Lecture 10

Next Lecture :14th September (Friday)
2:30-4:00

No Lecture on 13th September (Holiday)