Electrodynamics and Radiative Processes I

Lecture 1 – Radiation & Radiative Transfer

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Radiation

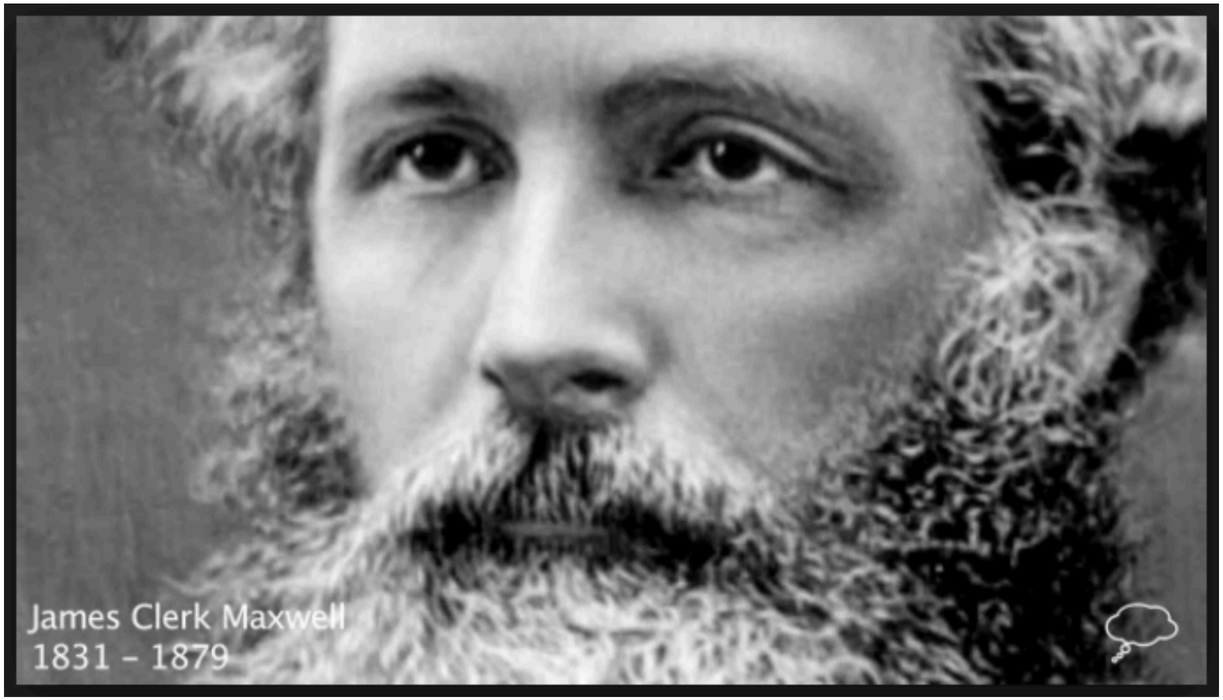
Observational astrophysics is study of radiation received from astronomical body

Important to learn about production of radiation
and interaction of radiation with matter

Radiation travelling through matter gets modified:

a) Absorbed
b) Scattered
c) Cause stimulated emission
d) Get modified by spontaneous emission AND MORE

Electromagnetic wave



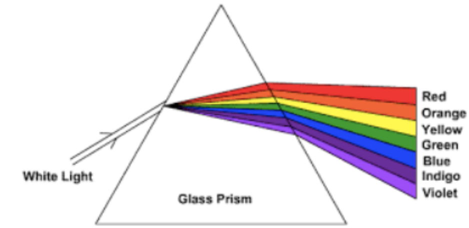
Existence of electromagnetic wave predicted by **Maxwell** confirmed by **Hertz**

↓

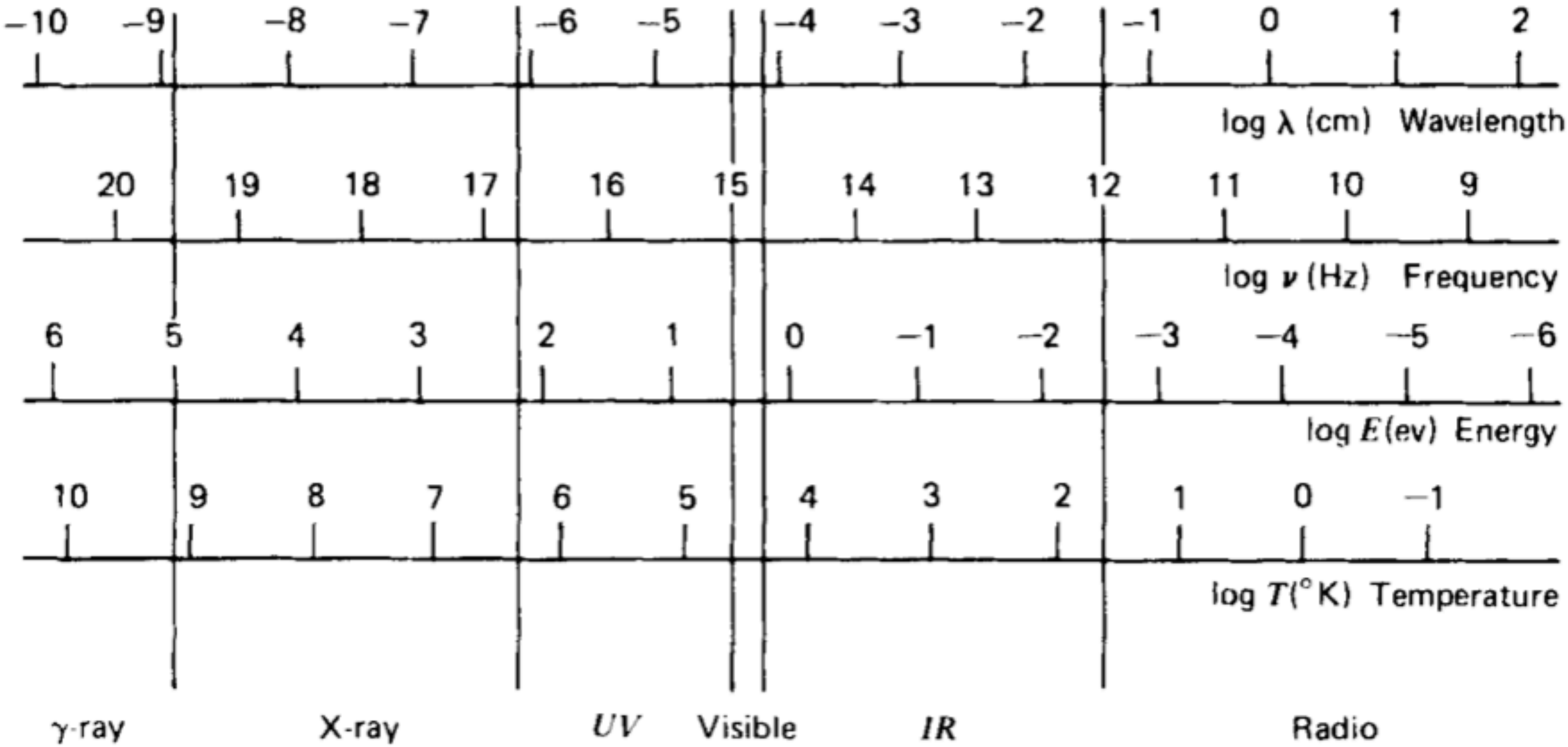
Experiments showed oscillating electric charge radiate electromagnetic wave.

Energy of EM waves is proportional to energy of oscillation of electric charge.

Electromagnetic spectrum

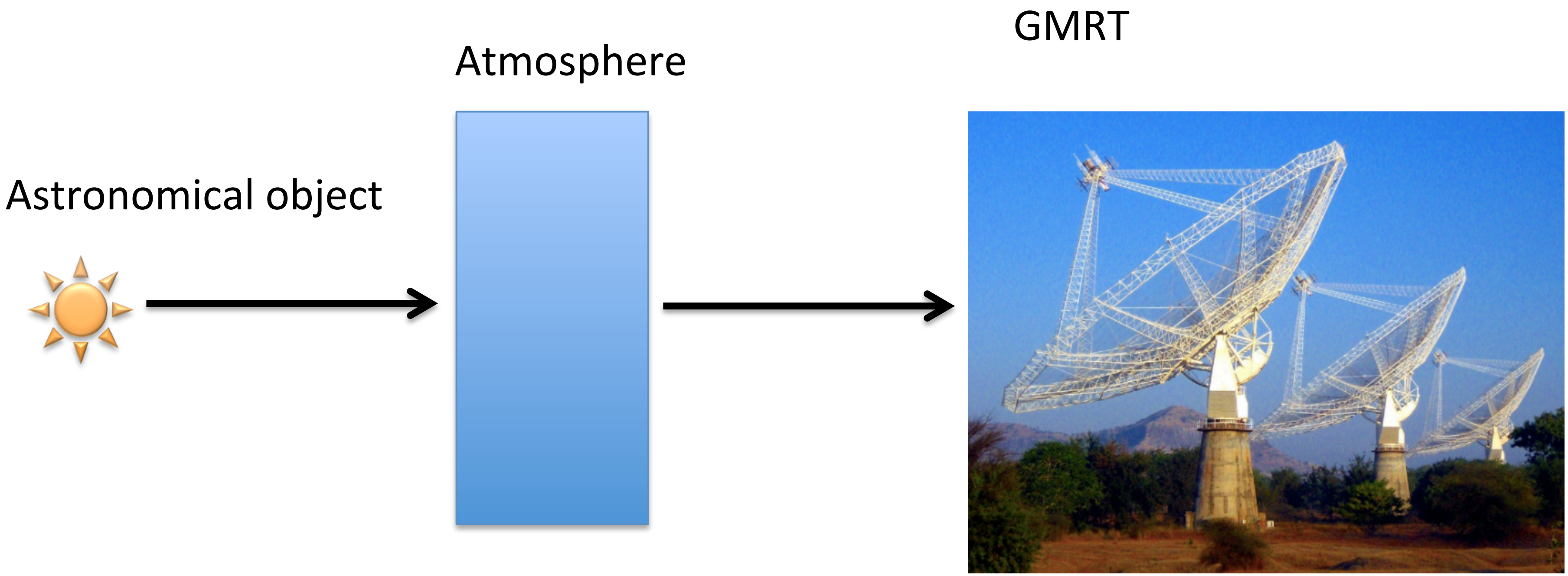


Electromagnetic radiation can be decomposed into spectrum.



$\lambda \nu = c$ $\quad E = h\nu$ $\quad T = E/k$

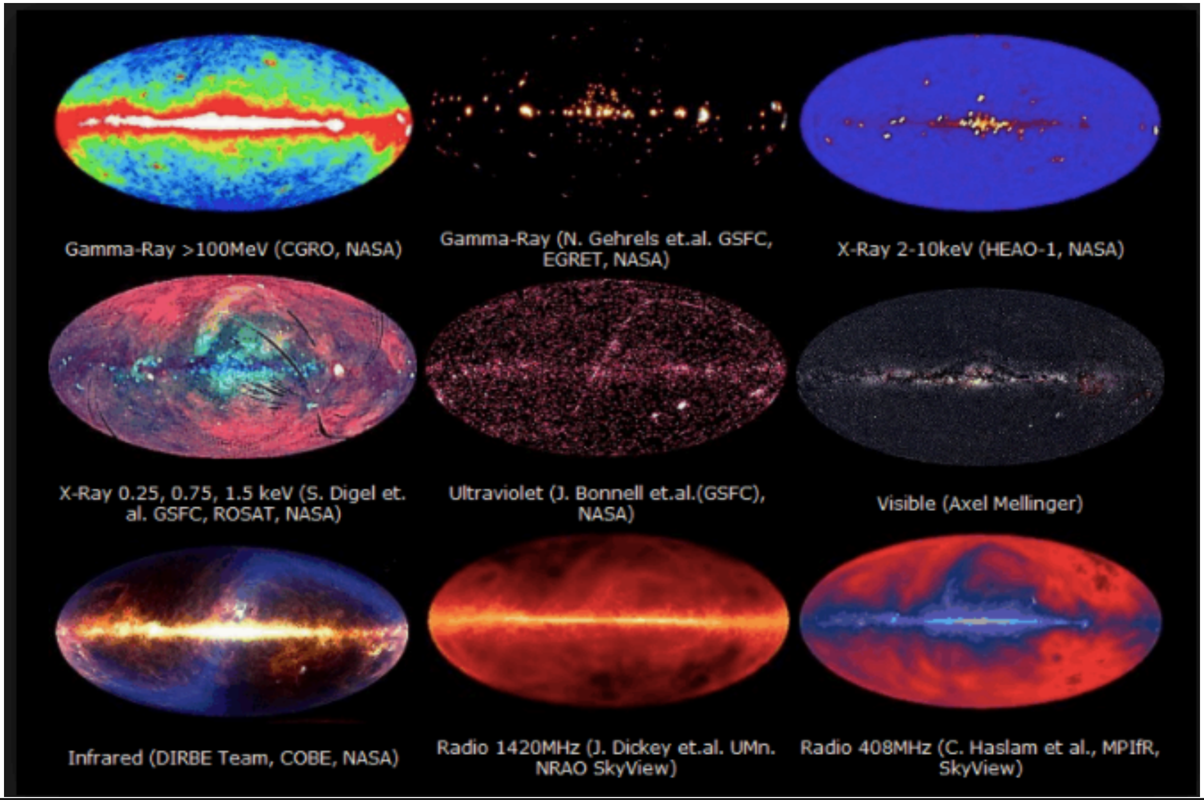
Radiation

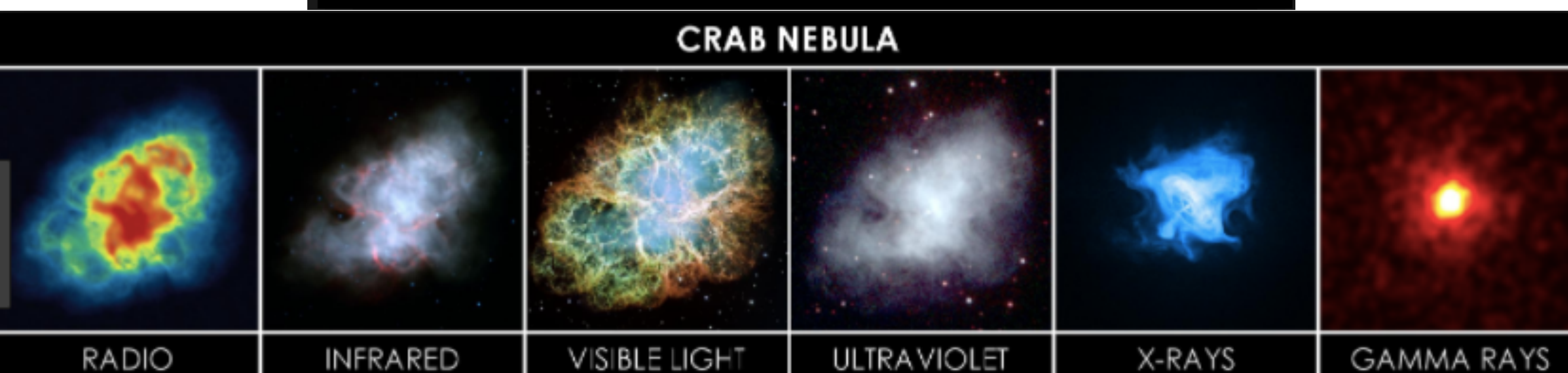


We are most sensitive to electromagnetic radiation.

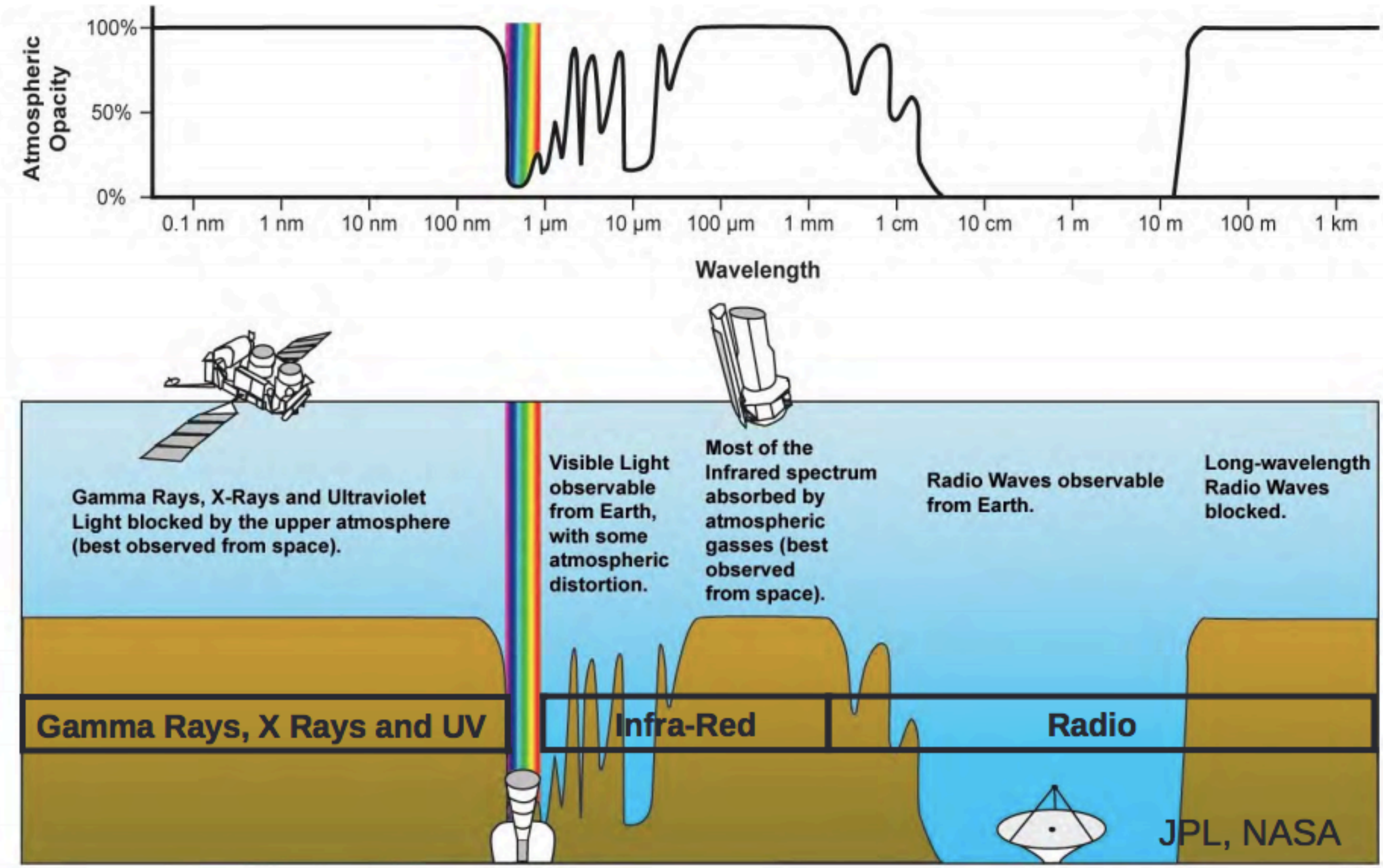
Radiation can be considered to travel in straight lines for practical purposes.

Radiation


Gamma-Ray >100MeV (CGRO, NASA)
Gamma-Ray (N. Gehrels et.al. GSFC, EGRET, NASA)
X-Ray 2-10keV (HEAO-1, NASA)
X-Ray 0.25, 0.75, 1.5 keV (S. Digel et. al. GSFC, ROSAT, NASA)
Ultraviolet (J. Bonnell et.al.(GSFC), NASA)
Visible (Axel Mellinger)
Infrared (DIRBE Team, COBE, NASA)
Radio 1420MHz (J. Dickey et.al. UMn. NRAO SkyView)
Radio 408MHz (C. Haslam et al., MPIfR, SkyView)


CRAB NEBULA
RADIO
INFRARED
VISIBLE LIGHT
ULTRAVIOLET
X-RAYS
GAMMA RAYS

Radiation received at Earth



Specific Intensity or Brightness I_ν

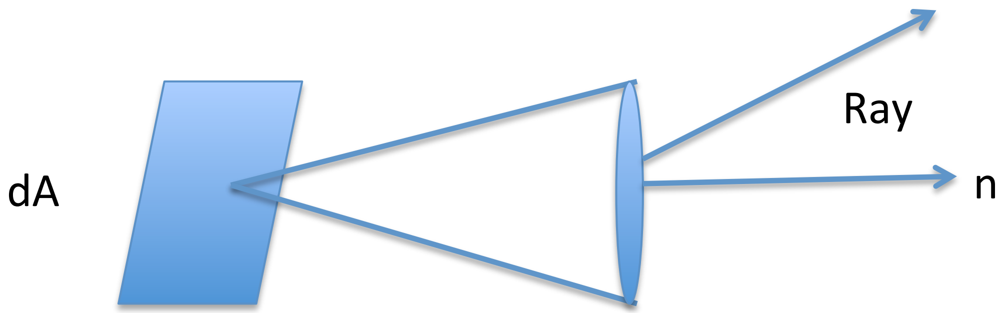


Figure: Geometry for normal incidence

Energy crossing dA in time dt in frequency range dν and into a solid angle dΩ

$$dE = I_\nu \, dA \, dt \, d\Omega \, d\nu$$

Specific Intensity or Brightness

$[I_\nu]$= erg cm^{-2} s^{-1} Hz $^{-1}$ ster^{-1}

Brightness does not decrease with distance

Specific Intensity or Brightness I_ν

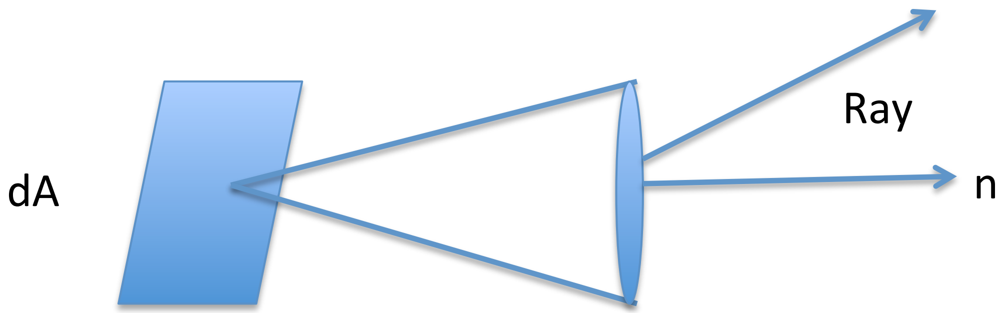


Figure: Geometry for normal incidence

Energy crossing dA in time dt in frequency range $d\nu$ and into a solid angle $d\Omega$

$$dE = F\, dA\, dt$$

Energy flux

$[F] = \text{erg cm}^{-2}\,\text{s}^{-1}$

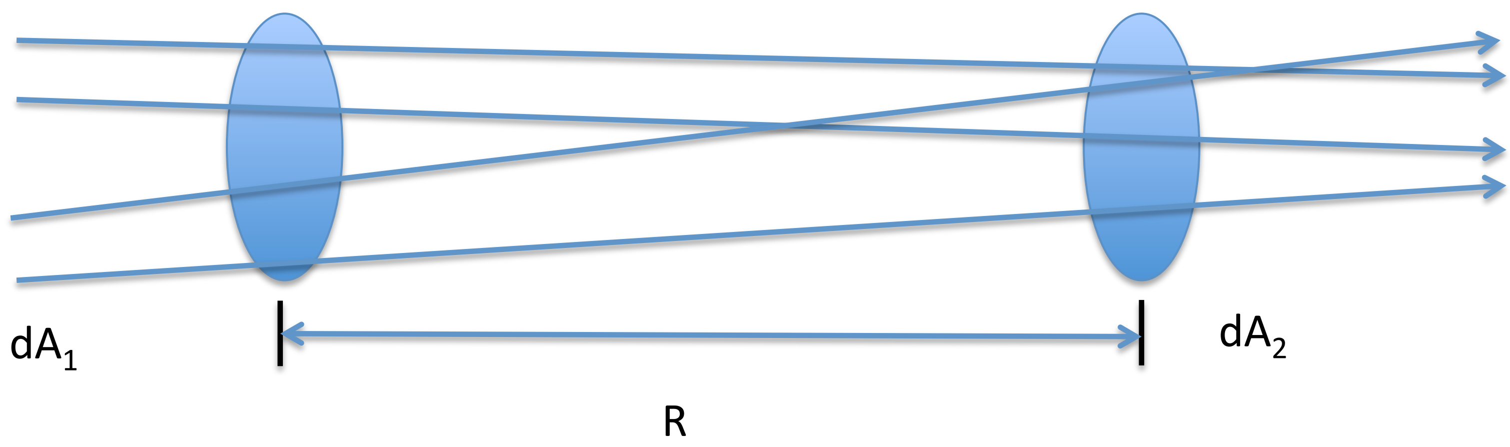
$$dE = I_\nu\, dA\, dt\, d\Omega\, d\nu$$

Specific Intensity or Brightness

$[I_\nu] = \text{erg cm}^{-2}\,\text{s}^{-1}\,\text{Hz}^{-1}\,\text{ster}^{-1}$

Brightness does not decrease with distance

Specific Intensity constant across a ray



$$dE_1 = I_{\nu_1} dA_1 dt\, d\Omega_1 d\nu_1 = dE_2 = I_{\nu_2} dA_2 dt\, d\Omega_2 d\nu_2.$$

$$d\Omega_1 = dA_2 / R^2,\ d\Omega_2 = dA_1 / R^2$$

$$I_{\nu_1} = I_{\nu_2}.$$

Specific Intensity for oblique incidence

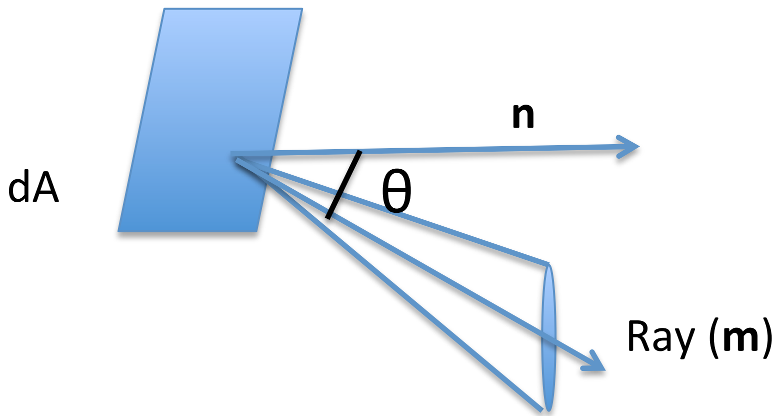


Figure: Geometry for oblique incidence

Flux through area dA (orientation n) and incident ray (orientation m) will be reduced by a factor of n.m=cosθ

$$dF_\nu(\text{erg s}^{-1}\text{ cm}^{-2}\text{ Hz}^{-1}) = I_\nu \cos\theta \, d\Omega.$$

Net flux

$$F_\nu = \int I_\nu \cos\theta \, d\Omega.$$

Net flux is zero for isotropic radiation

Total flux

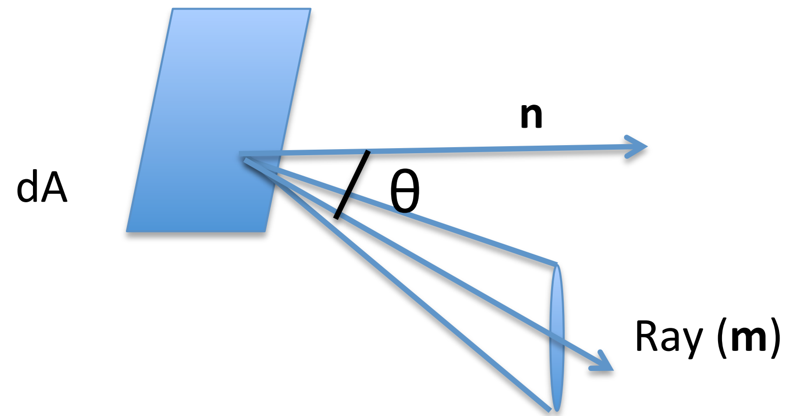


Figure: Geometry for oblique incidence

Integrate over frequency to get the total flux

$$F(\mathrm{erg\ s^{-1}\ cm^{-2}}) = \int F_\nu \, d\nu$$

Integrate over frequency to get the total Intensity

$$I(\mathrm{erg\ s^{-1}\ cm^{-2}\ ster^{-1}}) = \int I_\nu \, d\nu$$

Specific Intensity and total intensity

- ✓ Intrinsic property of the source
- ✓ Independent of the distance from the source
- ✓ Can be thought of as energy received at the detector OR as energy emitted by the source.

Total Intensity - Specific Intensity integrated over frequency

$$I(\mathrm{erg\ s^{-1}\ cm^{-2}\ ster^{-1}}) = \int I_\nu d\nu$$

Flux density - Total spectral power received from a source by a detector of unit area

Flux density - Dependent on distance to the source

Specific energy density
Radiative energy density

Specific energy density u_ν is defined as energy per unit volume per unit frequency range.

Let us first consider $u_\nu(\Omega)$; $dE = u_\nu(\Omega) dV\, d\Omega\, d\nu$; $dV = c\, dt\, dA$

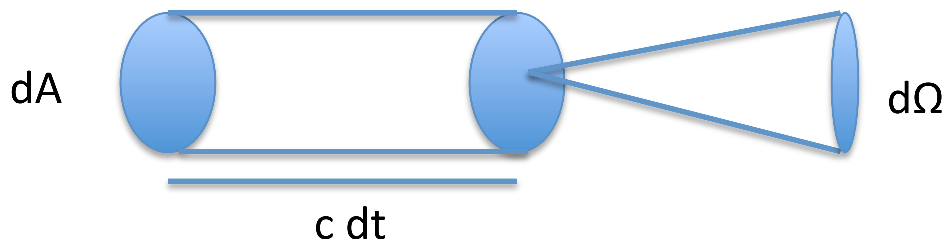


Figure: Electromagnetic energy in a cylinder

Energy crossing dA in time dt in frequency range dν and into a solid angle dΩ

$$dE = u_\nu(\Omega)\, dA\, c\, dt\, d\Omega\, d\nu.$$

Specific energy density
Radiative energy density

Specific energy density u_ν is defined as energy per unit volume per unit frequency range.
Let us first consider $u_\nu(\Omega)$; $dE = u_\nu(\Omega) dV\, d\Omega\, d\nu$; $d\nu = c\, dt\, dA$

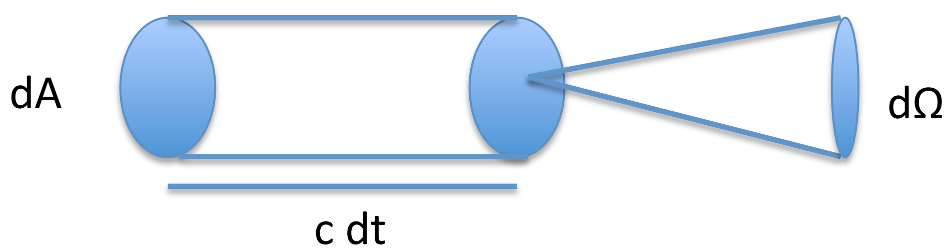


Figure: Electromagnetic energy in a cylinder

Energy crossing dA in time dt in frequency range dν and into a solid angle dΩ

$$dE = u_\nu(\Omega)\, dA\, c\, dt\, d\Omega\, d\nu. \quad \Longleftrightarrow \quad dE = I_\nu\, dA\, dt\, d\Omega\, d\nu$$

↓ From slide 8

$$u_\nu(\Omega) = \frac{I_\nu}{c}$$

Specific energy density
Radiative energy density

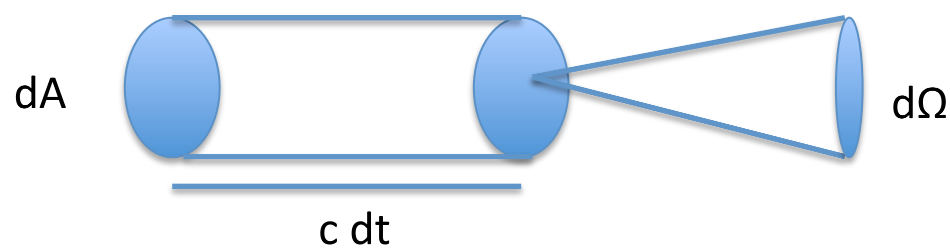


Figure: Electromagnetic energy in a cylinder

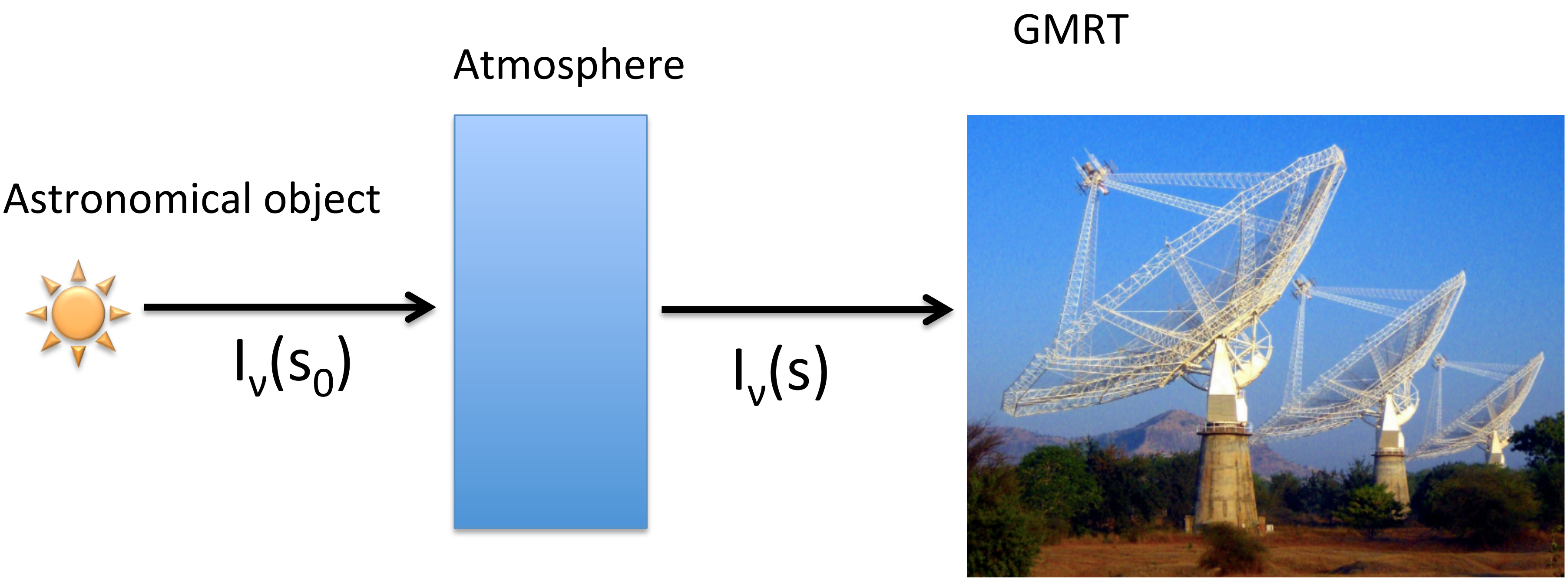
$$u_\nu(\Omega) = \frac{I_\nu}{c} \longrightarrow u_\nu = \int u_\nu(\Omega)\, d\Omega = \frac{1}{c}\int I_\nu\, d\Omega,$$

Mean Intensity

$$J_\nu = \frac{1}{4\pi}\int I_\nu\, d\Omega$$

$$u_\nu = \frac{4\pi}{c} J_\nu$$

Radiative transfer

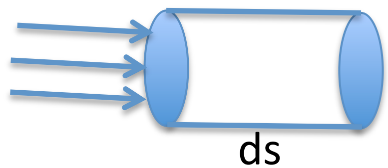


Radiative transfer is phenomenon of energy transfer from electromagnetic radiation.

Radiative Transfer
Emission Coefficient j_ν

If a ray passes through matter energy may be added or subtracted from it and specific intensity will not be constant : Emission and Absorption

Considering only Emission



$$dE = j_\nu \, dV \, d\Omega \, dt \, d\nu,$$

Spontaneous emission coefficient :
Energy emitted per unit time per unit solid angle per unit volume

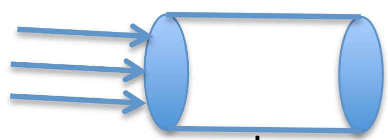
$[j_\nu]$= erg cm^{-3} s^{-1} Hz^{-1} Sr^{-1}

Specific Intensity added to the beam by spontaneous emission

$$dI_\nu = j_\nu \, ds$$

Radiative Transfer
Emission Coefficient j_ν

Considering only Emission



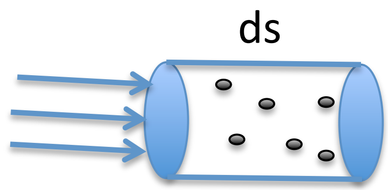
$$dE = j_\nu \, dV \, d\Omega \, dt \, d\nu, \quad \longleftrightarrow \quad dE = I_\nu \, dA \, dt \, d\Omega \, d\nu$$

$$dI_\nu = j_\nu \, ds.$$

Specific Intensity added to the beam by spontaneous emission

Absorption Coefficient α_ν

Loss of brightness in a beam as it travels a distance ds is



$$dI_\nu = -\alpha_\nu I_\nu ds. \quad \text{(4)}$$

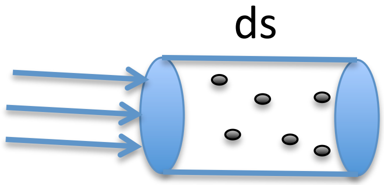
Absorption coefficient

$[\alpha_\nu]$= cm^{-1}

Consider n particles per unit volume each with cross-section σ_ν

$$\alpha_\nu = n\sigma_\nu.$$

Radiative Transfer Equation



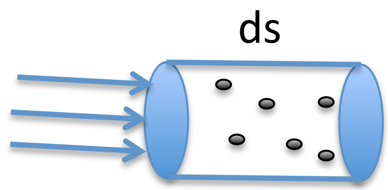
$$dI_\nu = j_\nu \, ds$$

$$dI_\nu = -\alpha_\nu I_\nu \, ds$$

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

Fundamental equation of Radiative transfer

Radiative Transfer Equation



$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

Emission only

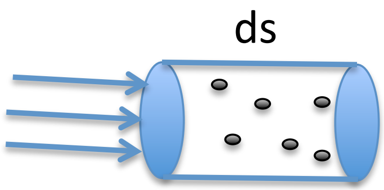
$$\frac{dI_\nu}{ds} = j_\nu$$

Solution

$$I_\nu(s) = I_\nu(s_0) + \int_{s_0}^{s} j_\nu(s')\,ds'$$

Increase in brightness = emission coefficient integrated across line of sight

Radiative Transfer Equation



$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

Absorption only

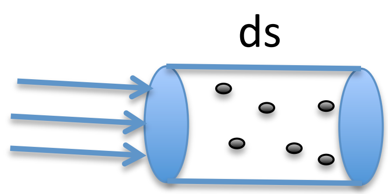
$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu$$

Solution

$$I_\nu(s) = I_\nu(s_0)\exp\left[-\int_{s_0}^{s}\alpha_\nu(s')\,ds'\right]$$

Decrease in brightness along the ray by exponential of absorption coefficient integrated across line of sight

Radiative Transfer Equation



$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

Emission only

$$\frac{dI_\nu}{ds} = j_\nu \quad \xrightarrow{\text{Solution}} \quad I_\nu(s) = I_\nu(s_0) + \int_{s_0}^{s} j_\nu(s')\, ds'$$

Absorption only

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu \quad \xrightarrow{\text{Solution}} \quad I_\nu(s) = I_\nu(s_0) \exp\left[-\int_{s_0}^{s} \alpha_\nu(s')\, ds' \right]$$

End of Lecture 1

Next lecture : 7th August