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Instability of a convection zone with large gradient of the mean molecular weight

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Abstract. An inefficient convective solution with non - negligible gradient of the mean molecular weight was discovered by extending the mixing-length theory of convection (Umezu & Nakakita 1988). This solution is unstable, but becomes stable if the radiative heat loss from the convective element is negligible. However, since the size of the convective element necessary to stabilize this solution is larger than the stellar radius by several orders, this inefficient convection solution never becomes stable. If the radiative heat loss is neglected, the zone with this inefficient convection solution resembles the Ledoux semiconvection zone. Hence this study might show another reason against the Ledoux semiconvection zone.

Keywords : stars: interiors

1. Introduction

The mixing-length theory of convection (Vitense 1953; Böhm-Vitense 1958;Cox & Giuli, 1968) was extended so as to include effects of a gradient of the mean molecular weight(μ-gradient, ∇_μ) or helium flux (Umezu & Nakakita 1988). It is found that there are two solutions in the extended mixing - length theory of convection. In addition to the convective solution with adiabatic temperature gradient($\nabla_{\rm ad}$) and with negligible μ-gradient, there is another solution with almost radiative temperature gradient($\nabla_{\rm rad}$) and with non-negligible μ-gradient. The latter solution expresses an inefficient convective solution. The

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radiative heat loss from the convective element of the inefficient solution is very significant.

Eggleton (1983) proposed the time-dependent theory of convection with μ-gradient. As its stationary solutions, the above-mentioned convective solutions can be reproduced (Nakakita & Umezu 1994). With Eggleton's theory, the linear stability analysis of the convective solutions is possible. In the previous work (Nakakita & Umezu 1994) it is suggested that the convective solution with large μ-gradient is unstable when the mixing-length is of the order of the pressure scale height.

Unstable condition in stellar interior is determined by considering a convective element displaced from the equilibrium position. If the convective element continues to move gaining energy from buoyancy, the place is unstable. For chemically inhomogeneous matter, the buoyancy is proportional to the density difference between the convective element and the environment. Hence the neutral condition is zero density difference. It becomes the Ledoux neutral condition (Ledoux 1947; Sakashita & Hayashi 1959) if the radiative heat loss from the convective element is neglected. Thus the mixing-length theory of convection is closely related to the Ledoux's neutral condition. Kato (1966), however, indicated that the radiative heat loss from the convective element destabilizes the zone with the Ledoux neutral condition (the Ledoux neutral zone). In this case, the state is overstable, i.e., perturbations grow vibrationally and the neutral condition is given by zero temperature difference i.e. the Schwarzschild neutral condition (Schwarzschild & Härm 1958).

Schwarzschild & Härm(1958) encountered the semiconvection problem when they constructed massive main sequence star models. When the opacity is due to electron scattering, the convective core grows as the star models evolve. In the convection zone, since chemical elements are transported very efficiently, they are distributed uniformly in the convective region. As the star evolves, hydrogen in the convective core decreases and the radiative temperature gradient also decreases. So the inner side of the convective core boundary becomes radiative. This is a contradiction. Hence the zone of varying chemical abundances must be introduced. This zone is called semiconvection zone. Which neutral condition is appropriate in the semiconvection zone of massive stellar models (more massive than about $10M_{\odot}$) is the so-called semiconvection problem. Various models have been calculated and discussed up to the present (see, for example, Stothers 1970; Langer et al. 1983; Grossman, Narayan & Arnett 1993; Grossman & Taam 1996; Canuto 1999, 2000; Stothers & Chin 2000). Almost all the authors think the semiconvection zone as an overstable zone and prefer to the Schwarzschild neutral condition (for example, Straniero et al. 2003; Poelarends et al. 2008).

For a formation of the Ledoux neutral zone, it is necessary that convective motion becomes self-destructive: convective velocity approaches zero and the μ-gradient becomes gradually larger. Thereby it is assumed that large μ-gradient can exist in convection zone.

In order to examine this assumption, both the radiative heat loss and the μ-gradient are to be considered. Eggleton's equations are appropriate to the study of this problem.

In this work, it is indicated that the Ledoux neutral zone cannot be interpreted as an inefficient convection zone. Hence it probably provides another reason against the Ledoux neutral zone. In Section 2, the radiative heat loss is estimated by assuming that the size of the convective element is different from the mixing-length in order to investigate the effect of the radiative heat loss irrespective of the mixing-length. Thereby the equation for the entropy difference in the Eggleton's equations is slightly modified. Numerical results of massive stars are presented in Section 3 as the semiconvection is traditionally a massive star problem. However it can also be related to intermediate low-mass models (Popielski & Dziembowski 2005). The concluding remarks and brief comments on the semiconvective problem are given in Section 4. The semiconvection has been traditionally considered without rotation and magnetism. Hence in this work they are neglected though they might play a role in the semiconvection problem.

2. Basic equations

2.1 The radiative heat loss from the convective element of size $\ell_{\rm b}$

We estimate the radiative heat loss from the convective element of size $\ell_{\rm b}$, volume V and surface area A. A temperature difference between the convective element and the surrounding medium is denoted as δT. The size of the convective element $\ell_{\rm b}$ is assumed to be different from the mixing-length ℓ that is the distance from the place of birth of the convective element to the place of its dissociation. We assume that the convective element is optically thick since we consider the convection in stellar interior. The radiative heat flux $F_{\rm r}$ per unit time per unit area can be written as

$$F_r = -C_{\rm P}\rho\chi\frac{{\rm d}T}{{\rm d}r}, \quad \text{where} \quad \chi = \frac{16\sigma T^3}{3\kappa\rho^2 C_P} \tag{1}$$

where C_p is the specific heat per unit mass at constant pressure and ρ is the density; and σ is the Stefan-Boltzmann constant; and κ is the opacity. The radiative heat loss from the surface of the convective element per unit time per unit mass ($q_{\rm loss}$) can be estimated as

$$q_{\rm loss} = C_P\chi\frac{A}{V}\frac{\delta T}{\ell_{\rm b}} = \frac{3}{2}C_P\chi\frac{\delta T}{y\ell_{\rm b}^2}, \quad \text{where} \quad \frac{1}{y} = \frac{2A\ell_{\rm b}}{3V}. \tag{2}$$

This parameter y is concerned only with the convective element i.e. its shape, its internal temperature distribution and so on (Henyey et al. 1965). The entropy loss $S_{\rm loss}$ becomes, dividing $q_{\rm loss}$ by T,

$$S_{\rm loss} = \frac{C_P\delta T}{\gamma_{\rm b}\ell T}, \quad \text{where} \quad \gamma_{\rm b} = \frac{2}{3\chi}\frac{\ell_{\rm b}^2}{\ell}y. \tag{3}$$

This equation is used in the right-hand side of Eggleton's equation for the entropy difference δS, Eq(7), below.

The timescale for the radiative heat loss $\tau_{\rm rad}$ and the life-time of the convective element $\tau_{\rm life}$ with the velocity δv can be expressed as

$$\tau_{\rm rad} = \frac{C_P \delta T}{q_{\rm loss}} \quad \text{and} \quad \tau_{\rm life} = \frac{\ell}{\delta v}, \tag{4}$$

respectively. Thereby the ratio of these timescales (γ) can be written as

$$\gamma = \gamma_{\rm b} \delta v = \frac{C_P \delta T}{q_{\rm loss}} \frac{\delta v}{\ell} = \frac{\tau_{\rm rad}}{\tau_{\rm life}}. \tag{5}$$

If there is enough time for the radiation to escape from the convective element during the existence of the convective element ($\tau_{\rm rad} \ll \tau_{\rm life}, \gamma \ll 1$), the radiative heat loss cannot be neglected. On the other hand, if the convective element disappears before radiation escapes from the convective element ($\tau_{\rm rad} \gg \tau_{\rm life}, \gamma \gg 1$), the radiative heat loss can be neglected. Hence γ represents the degree of the radiative heat loss (See Fig. 2). The larger the convective element is, the less significant the radiative heat loss is. If ℓ is smaller, the lifetime of the convective element becomes probably smaller. Then the radiative heat loss has shorter time to affect the temperature difference of the convective element. Hence we can change the degree of the radiative heat loss artificially by changing $\ell_{\rm b}$ and ℓ.

2.2 Equations for the mixing-length theory of convection

Differences in the mean molecular weight, entropy and velocity between the convective element and the surroundings are denoted by $\delta\mu$, δS and δv, respectively. Using Eq(3), the basic equations for convection are (Eggleton 1983; Nakakita & Umezu 1994);

$$\frac{\partial \delta \ln \mu}{\partial t} + \delta v \left(-\frac{\nabla_\mu}{H_P} + \frac{\delta \ln \mu}{\ell} \right) = 0, \tag{6}$$

$$\frac{\partial \delta S}{\partial t} + \delta v \left(-\frac{dS}{d \ln P} \frac{1}{H_P} + \frac{\delta S}{\ell} \right) = -\frac{\alpha_{\rm r}}{\gamma_{\rm b} \ell} \left[\delta S - \left(\frac{\partial S}{\partial \ln \mu} \right)_{P,T} \delta \ln \mu \right], \tag{7}$$

$$\frac{\partial \delta v}{\partial t} + \frac{(\delta v)^2}{\ell} = \frac{T \nabla_{\rm ad}}{C_3 H_P} \left[\delta S - \left(\frac{\partial S}{\partial \ln \mu} \right)_{P,\rho} \delta \ln \mu \right]. \tag{8}$$

where P is the pressure and H_p is the pressure scale height;C_3 is a parameter concerned with viscous dissipation. The right-hand side of (7) represents the radiative heat loss from the perturbation as mentioned above. A new parameter $\alpha_{\rm r}$ is introduced: $\alpha_{\rm r} = 1$ for Eggleton's theory: if $\alpha_{\rm r} = 0$, the radiative heat loss can be neglected completely.

The total heat flux F, the radiative heat flux $F_{\rm r}$ and the convective heat flux $F_{\rm c}$ are given as follows:

$$F = \rho C_P T \chi \frac{1}{H_P} \nabla_{\rm rad}, \quad F_{\rm r} = \rho C_P T \chi \frac{1}{H_P} \nabla, \quad \text{and} \quad F_c = C_1 \rho C_P T \delta v \delta \ln T \tag{9}$$

where C_1 is a parameter of the order of one.

The last equation in (9) is written according to the Haase's definition of heat. The heat flux formula with the Prigogine's definition of heat was also considered in the previous work (Nakakita & Umezu 1994). Under the condition that heat is related to warming and cooling of material, it is peculiar that heat can be created only by mixing chemical species with constant pressure and temperature (Haase 1969; Smith 1980; Canuto 1999). Hence the Haase's definition of heat is used throughout this work. For these fluxes, the equation

$$F = F_r + F_c \tag{10}$$

holds.

With these equations we obtain the following equations for steady convection (See for details Eggleton 1983; Nakakita & Umezu 1994);

$$\nabla = \frac{(\alpha_{\rm r} + \gamma)\nabla_{\rm rad} + \phi\gamma^2\nabla_{\rm ad}}{\alpha_{\rm r} + \gamma + \phi\gamma^2}, \tag{11}$$

$$\phi\gamma^4 + \gamma^3 + \left(\alpha_{\rm r} + \frac{\alpha {\rm g}}{C_3}\gamma_{\rm b}^2\ell\phi\nabla_\mu Q_\mu\right)\gamma^2 - \frac{\alpha {\rm g}}{C_3}\gamma_{\rm b}^2\ell\left[Q(\nabla_{\rm rad} - \nabla_{\rm ad}) - Q_\mu\nabla_\mu\right]\gamma + \frac{\alpha {\rm g}}{C_3}\gamma_{\rm b}^2\ell Q_\mu\nabla_\mu\alpha_{\rm r} = 0, \tag{12}$$

where

$$\alpha = \frac{\ell}{H_P}, \quad Q = -\left(\frac{\partial \ln\rho}{\partial \ln T}\right)_{P,\mu}, \quad Q_\mu = \left(\frac{\partial \ln\rho}{\partial \ln \mu}\right)_{P,T}, \phi = \frac{C_1\ell}{\chi\gamma_{\rm b}} \tag{13}$$

and g is the gravitational acceleration. Since the pressure is the sum of the gas pressure and the radiation pressure in the present study, $Q_\mu = 1$. Eq (12) is at a glance quite strange: convective velocity becomes zero only if $\nabla_\mu = 0$. Hence the μ-gradient must be zero at the convective core boundary. This condition is different from both of the Ledoux neutral condition ($\nabla_{\rm rad} - \nabla_{\rm ad} - \nabla_\mu/Q = 0$) and the Schwarzschild neutral condition ($\nabla_{\rm rad} - \nabla_{\rm ad} = 0$). These conditions are proposed as the conditions with which the convective velocity is zero. The condition that $\nabla_\mu = 0$ at the convective core boundary is, however, in accordance with Cox & Giuli's analysis of semiconvection zone (Cox & Giuli, 1968). They stated as follows: suppose that the convective element stops at some place within the Ledoux neutral zone. Since in the frame work of local theory, this means that buoyancy becomes zero, i.e., $\delta\ln\rho = -Q\delta\ln T + \delta\ln\mu = 0$. The temperature difference $\delta\ln T$ gradually decreases due to the radiative heat loss while $\delta\ln\mu$ does not. Hence the density of the convective element becomes larger than the surroundings. Thereby the convective element begins to move downwards. This means that the Ledoux condition is not a real neutral condition. Both the conditions $\delta\ln\rho = 0$ and $\delta\ln\mu = 0$ are necessary for the strict neutral state. The condition that $\delta\ln\mu = 0$ leads to the condition that ∇_μ=0 from Eq. (6) for the steady convection.

It can also be explained that the radiative heat loss from the convective element reduces the μ-gradient in the steady convection(Umezu & Nakakita 1988, See Fig. 4 below). If the neutral condition $\delta \ln \rho = -Q\delta \ln T + \delta \ln \mu = 0$ is completely valid, the μ-gradient is given by $\nabla_\mu = Q\delta \ln T/\alpha$. When the temperature difference is reduced due to the radiative heat loss, the μ-gradient also becomes smaller.

Helium flux per unit time per unit area F_Y is given by, with a parameter C_2 of the order of one,

$$F_Y = -C_2 \rho \delta v \ell \frac{dY}{dr} \tag{14}$$

and related to the μ-gradient as

$$\nabla_\mu = \phi_Y \frac{\gamma_{\rm b} F_Y}{\alpha\gamma}, \quad \text{where} \quad \phi_Y = \frac{5\mu}{4C_2\rho}. \tag{15}$$

Eq (12) can be expressed, by using F_Y instead of ∇_μ, as

$$\phi\gamma^5 + \gamma^4 + \alpha_{\rm r}\gamma^3 - \frac{\mathrm{g}}{C_3}\gamma_{\rm b}^2 \ell \left[-\gamma_{\rm b}\phi\phi_Y F_Y Q_\mu + \alpha Q(\nabla_{\rm rad} - \nabla_{\rm ad})\right]\gamma^2$$

$$+\frac{\mathrm{g}}{C_3}\gamma_{\rm b}^3 \ell Q_\mu \phi_Y F_Y \gamma + \frac{\mathrm{g}}{C_3}\gamma_{\rm b}^3 \ell Q_\mu \phi_Y \alpha_{\rm r} F_Y = 0. \tag{16}$$

As is shown in Umezu & Nakakita(1988) and Nakakita & Umezu(1994), this equation has two convective solutions. One solution has almost adiabatic temperature gradient and negligible μ-gradient and is in accord with the ordinary notion of convection. Another solution has almost radiative temperature gradient and non-negligible μ-gradient. This latter solution expresses an inefficient convection.

2.3 Equations for the linear stability analysis

In this subsection the quantities of steady solution are denoted with subscript $_0$, and the time-dependent perturbations are denoted with Δ before the quantity as

$$\delta \ln \mu = \delta \ln \mu_0 + \Delta \ln \mu, \quad \delta S = \delta S_0 + \Delta S \quad \text{and} \quad \delta v = \delta v_0 + \Delta v. \tag{17}$$

Assuming that the time dependence of the perturbations is as $e^{\lambda t}$, the equation for the growth rate (λ) is obtained:

$$\left(\lambda + \frac{\delta v_0}{\ell}\right)\left[\lambda^2 + \left(\frac{3\delta v_0}{\ell} + \frac{\alpha_{\rm r}}{\ell\gamma_{\rm b}}\right)\lambda + \frac{3(\delta v_0)^2}{\ell^2} + \frac{2\alpha_{\rm r}\delta v_0}{\ell^2\gamma_{\rm b}} - \frac{P}{C_3\rho H_P^2}(\nabla_\rho - \nabla_{\rho,\rm ad})\right] = 0. \tag{18}$$

If the radiative heat loss from the convective element is negligible($\alpha_{\rm r} = 0$), Eq(18) becomes

$$\left(\lambda + \frac{\delta v_0}{\ell}\right)\left[\lambda^2 + \frac{3\delta v_0}{\ell}\lambda + \frac{3(\delta v_0)^2}{\ell^2}. - \frac{P}{C_3\rho H_P^2}(\nabla_\rho - \nabla_{\rho,\rm ad})\right] = 0. \tag{19}$$

The first factor gives negative λ. The discriminant of the quadratic equation derived from the second braces is

$$D = -\frac{3(\delta v_0)^2}{\ell^2} + \frac{4P}{C_3 \rho H_P^2}(\nabla_\rho - \nabla_{\rho,\mathrm{ad}}). \tag{20}$$

In the convection region, $\nabla_{\rho,\mathrm{ad}} - \nabla_\rho = Q(\nabla - \nabla_{\mathrm{ad}}) - \nabla_\mu$ is positive if the radiative heat loss is neglected. Hence the quadratic equation has complex roots whose real parts are negative($-3\delta v_0/2\ell$). Hence the steady solutions are all stable.

This artificial neglect of the radiative heat loss is also possible if the size of the convective element ℓ_b is infinite. The growth rate, as shown in Section 3, depends on the size of the convective element and the mixing-length. Quantitative studies with stellar models are necessary.

3. Numerical models

3.1 Numerical method

In this work 40M$_\odot$ models with initial composition $X = 0.70$ and $Z = 0.02$ are calculated. For simplicity, the followings are assumed. The ionization is complete. The stellar material is composed of hydrogen, helium and metal. Abundance of metal is constant throughout the star. Opacity is due to electron scattering. The gravitational energy release is negligible. Degeneracy is negligible. Nuclear energy generation rates are taken from Schwarzschild's text (Schwarzschild 1958).

The models are composed of three zones: the radiative envelope, the intermediate convection zone with large μ- gradient, and the central convection zone with negligible μ-gradient. The models are similar to the Ledoux's model and the Sakashita and Hayashi's model (Ledoux 1947; Sakashita & Hayashi 1959) if the radiative heat loss is negligible.

Previous calculations of stellar models with semiconvection zone indicate that the calculation by Henyey method is very difficult (Eggleton 1971, 1972; Grossman & Taam 1996). Hence the fitting method (for example, Sears & Brownlee 1965) is used. The equations for helium abundance (Y) and helium flux (L_Y) are added to the ordinary four differential equations of stellar structure (Umezu 1999). These equations are integrated inwardly from the stellar surface with given values of radius and luminosity. In the convective region, we first use the inefficient convective solution with large μ-gradient. From the place where helium abundance becomes prescribed central helium abundance, we use the efficient convective solution with negligible μ-gradient further until the prescribed fitting point is reached. The outward integrations of these equations from the centre are done with given values of central temperature, central pressure and central helium abundance. These five values at the surface and the centre are corrected until the differences of the physical quantities between these two integrations at the predetermined location become smaller than the prescribed values.

Table 1. Parameter values of the calculated models. The mixing length and a size of the convective element are ℓ and ℓ_b, respectively. H_p is the pressure scale height. ΔY_{ccb} is an artificial increment in helium abundance at the convective core boundary.

model	ℓ/H_P	ℓ_b/H_P	age(yrs)	ΔY_{ccb}
A	5.0×10^{-2}	1.0	1.90×10^5	1.0×10^{-2}
B	5.0×10^{-3}	1.0	3.17×10^5	1.0×10^{-2}
C	5.0×10^{-3}	5.0×10^3	4.75×10^5	1.0×10^{-2}
D	5.0×10^{-3}	5.0×10^5	8.24×10^5	1.0×10^{-2}
E	5.0×10^{-3}	5.0×10^5	3.49×10^5	1.0×10^{-4}

Umezu (1999) shows that the Ledoux neutral zone leads to the unphysical negative helium flux near the boundary between the radiative envelope and the neutral zone because $\nabla_\mu = 0$ at the boundary. With the mixing-length theory of convection presented here, μ-gradient must be zero at the core boundary (see Eq. (12)). Hence helium flux is thought to be negative in the present calculation similarly. In order to circumvent the problem of negative helium flux, helium content is artificially increased by amount of 10^{-2} at the convective core boundary except model E. In model E, helium abundance is increased by amount 10^{-4} in order to study the effect of this artificial increase in helium on the stability analysis. In the course of inward integrations, if the convective solution with large μ-gradient is not obtained due to negative helium flux near the convective core boundary, we proceed with the integrations assuming that the zone is radiative. Due to the insufficient increase in helium, the helium flux becomes negative in the course of the inward integration even if the helium flux is positive near the convective core boundary. Then the integration is stopped and the new parameter values are tried. The numerical values of parameters of the mixing-length theory of convection are: $C_1 = C_2 = 0.5, C_3 = 8, y = 0.076$ (Henyey et al. 1965).

3.2 Numerical results

The values of the size of the convective element ℓ_b and the mixing-length ℓ of the calculated models are cited in Table 1 in terms of the pressure scale height. ΔY_{ccb} is the above-mentioned artificial increment in helium abundance. If ℓ becomes smaller, or if ℓ_b becomes larger, the effects of the radiative loss become smaller. The effects of the radiative heat loss decrease from model A to model D as seen Fig. 2. In Table 2, luminosity (L), radius (R) and the effective temperature (T_e) of the models are given. The mixing length ℓ and the size of the convective element ℓ_b at the interface between the radiative envelope and the intermediate convection zone with large μ-gradient are also given in Table 2. For each combination of parameter values, only one evolutionary model is calculated. The numerical results are shown in the following figures.

Table 2. Characteristic values of the calculated models. L, R and T_e are the luminosity, the radius and the effective temperature of the model, respectively. The mixing length and a size of the convective element are ℓ and ℓ_b,respectively. $L_\odot$ and $R_\odot$ are the solar luminosity and the solar radius, respectively. The subscript $_{ccb}$ means the values at the interface between the radiative envelope and the intermediate convection zone with large μ-gradient.

model	$\log(L/L_\odot)$	$\log(R/R_\odot)$	$\log T_e$	$\log(\ell_b/R_\odot)_{ccb}$	$\log(\ell/R_\odot)_{ccb}$
A	3.858	0.9273	4.646	-0.117	-1.419
B	3.978	0.9342	4.646	-0.113	-2.414
C	4.019	0.9376	4.645	+3.588	-2.412
D	4.036	0.9420	4.643	+5.593	-2.408
E	3.962	0.9342	4.645	+5.586	-2.414

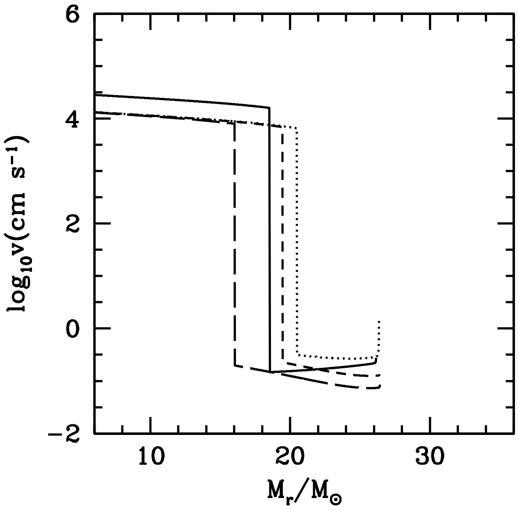

Figure 1. The convective velocities v in the convective region. The solid line, the dotted line, the dashed line and the long-dashed line represent convective velocities in models A,B,C and D, respectively. M_r is the mass coordinate and $M_\odot$ is the solar mass.

Fig. 1 shows the velocity distributions. In the chemically homogeneous central convective core, the convective element moves rapidly as fast as 10^4cm s^{-1}. On the other hand, in the intermediate convection zone with large μ-gradient, the convective velocity is as low as 10^{-1}cm s^{-1}. The convective velocity v in Fig. 1 is equal to the velocity difference δv in Section 2 since the velocity of the surroundings, i.e., the expansion or contraction velocity of whole star, is negligible.

In the chemically homogeneous core, convective velocity in model A is larger than in model B. The radiative heat loss from the convective element in the convective core of

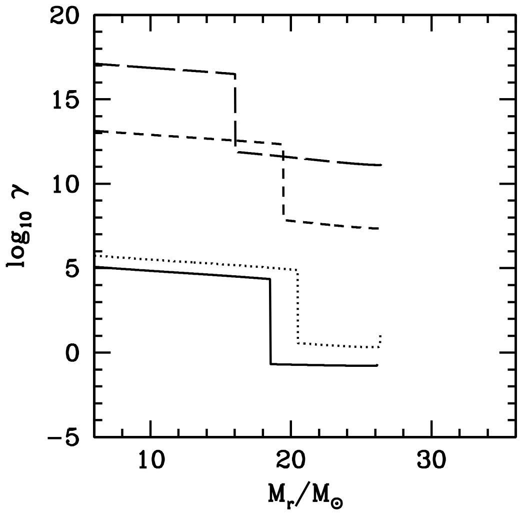

Figure 2. The radiative heat loss, γ, in the convective region. The notation is the same as Fig. 1.

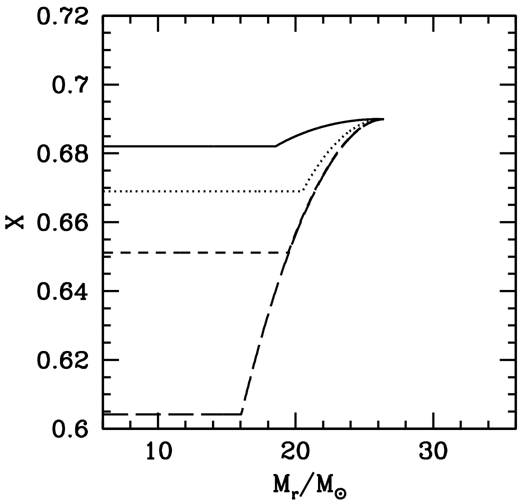

Figure 3. The hydrogen abundances of the models. The notation is the same as Fig. 1.

model A is greater than in the core of model B (Fig. 2). Hence the temperature difference $\delta \ln T$ in model A is smaller than in model B. The convective velocity is given by

$$(\delta v)^2 = \frac{1}{C_3} \ell g Q \delta \ln T, \tag{21}$$

in the central homogeneous convection zone. The larger mixing-length compensates for

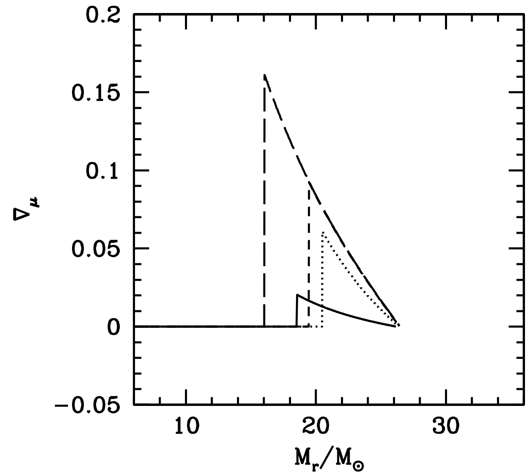

Figure 4. The μ- gradients in the convective region. The notation is the same as Fig. 1.

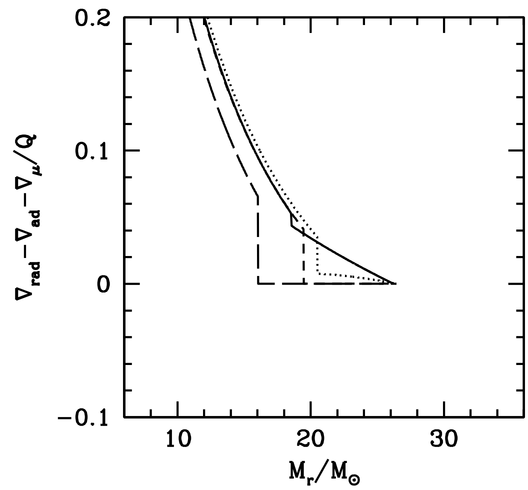

Figure 5. The values of $\nabla_{\rm rad} - \nabla_{\rm ad} - \nabla_\mu/Q$ in the convective region. The notation is the same as Fig. 1. The Ledoux neutral condition is $\nabla_{\rm rad} - \nabla_{\rm ad} - \nabla_\mu/Q = 0$.

the smaller temperature difference. On the other hand, in the intermediate chemically inhomogeneous zone, convective velocity is greater in model B than in model A. In this zone convective velocity is calculated according to

$$(\delta v)^2 = \frac{1}{C_3}\ell g(Q\delta \ln T - Q_\mu \delta \ln \mu). \tag{22}$$

The temperature difference in the intermediate zone of model A is also smaller than in model B. The mixing-length ℓ is concerned with the velocity as well as the radiative heat

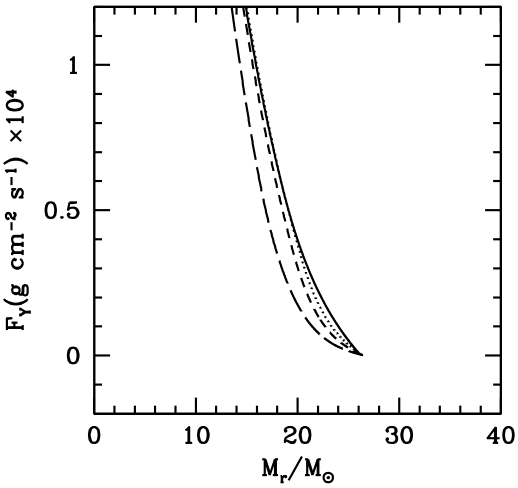

Figure 6. The helium flux F_Y per unit area. The notation is the same as Fig. 1

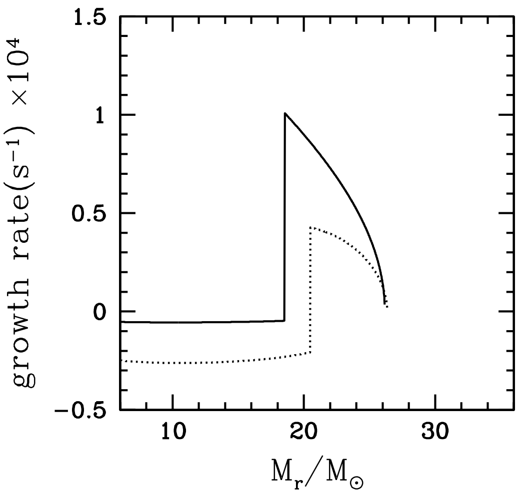

Figure 7. The growth rates multiplied by $10^4$of the convective solutions in models A and B. The notation is the same as Fig. 1.

loss. The complicated effect of $\delta \ln \mu$ reduces the density difference $Q\delta \ln T - Q_\mu \delta \ln \mu$ considerably: The larger mixing-length cannot compensate for it.

In Umezu & Nakakita(1988), the cases $\ell = \ell_\mathrm{b}$ and without the radiative heat loss (ℓ is finite and ℓ_b is infinite) are considered. In both cases, for a given values of helium flux, there are two roots for velocity. When the mixing-length becomes larger, the larger root

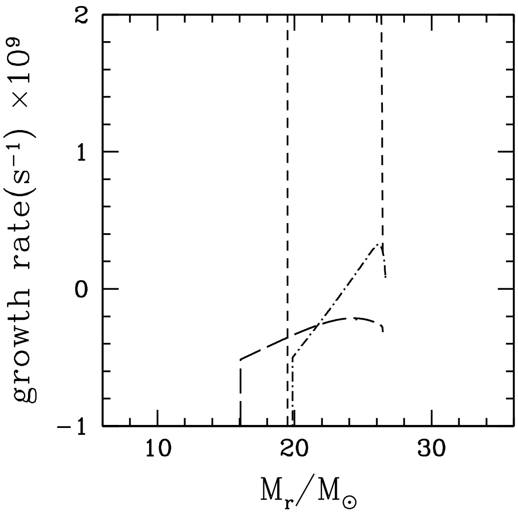

Figure 8. The growth rates multiplied by 10^9 of the convective solutions in models C, D and E. The dot-dashed line represents model E. The other notation is the same as Fig. 1.

of velocity becomes larger while the smaller root of velocity becomes smaller. This is one of the characteristics of the mixing-length theory of convection with helium flux.

From model A to model B, the velocity in the intermediate zone increases while from model B to model D, the velocity decreases. This different behaviour is due to the different parameter changes. From model A to B, ℓ is shortened while ℓ_b is the same. From model B to C, ℓ is the same while ℓ_b is increased. The size of the convective element ℓ_b is concerned only with the radiative heat loss. But the mixing-length ℓ influences the convective velocity as well as the radiative heat loss as mentioned above.

Fig. 2 shows that γ, as a whole, increases from model A to model D. Hence in this order the effects of the radiative heat loss decrease as explained in subsection 2.1. In models A and B the radiative heat loss is very significant in the intermediate zone since γ is as low as 10^{-2}. The opposite is true for models C and D.

Fig. 3 shows the hydrogen distributions. At the same mass coordinate in the intermediate zones, the gradients of the hydrogen distributions in models A and B are flatter than the gradients in the intermediate zones of models C and D. Reduction in μ- gradient of the intermediate zones at the same mass coordinate is also shown in Fig. 4. This is because the radiative heat loss reduces these gradients as explained in subsection 2.2.

In these figures, the intermediate convection zone in model A extends more than in model B. This is because the hydrogen gradient is smaller in model A than in model B. The age of the model is determined by the consumed amount of hydrogen. For a given age of the model, smaller gradient of hydrogen needs more extended intermediate convection zone to reach smaller amount of hydrogen in the central chemically homogeneous convective core and to compensate for larger amount of hydrogen in the intermediate convection zone.

In the intermediate zone, the very small buoyancy is necessary to maintain the very slow motion. The Ledoux neutral condition ($\nabla_{\rm rad} - \nabla_{\rm ad} - \nabla_\mu/Q = 0$) is valid in the intermediate convection zone of models C and D. However it is not valid in models A and B (Fig. 5). This deviation from the Ledoux neutral condition occurs due to the radiative heat loss since the Ledoux neutral condition is derived without considering the radiative heat loss.

Fig. 6 shows the helium flux per unit area (F_Y) multiplied by 10^4. A equation of helium flux through a sphere of radius r,$L_Y(= 4\pi r^2 F_Y)$, is given as,

$$\frac{dL_Y}{dM_r} = \epsilon_Y - \frac{Y - Y^n}{\Delta t},$$

where ϵ_Y is the helium generation rate; Y and Y^n are helium abundances of the present model and of the previous model,respectively; and Δt is the time interval between these models. This equation is integrated inward from the boundary between the intermediate convection zone and the envelope. In the radiative envelope $L_Y = 0$ and $dL_Y/dM_r = 0$. If the helium abundance is continuous across the boundary, $dL_Y/dM_r = 0$ at the inner side of the boundary. Since $L_Y = 0$ there, as shown in Umezu(1999), helium flux becomes negative in the Ledoux semiconvection zone and the inefficient convection solution does not exist. With the artificial increment in helium, which cannot be justified, $dL_Y/dM_r < 0$ at the inner side of the boundary, and the helium fluxes (L_Y and F_Y) increase inward and become positive as shown in Fig. 6. So other weak points of the Ledoux semiconvection zone can be studied with numerical models. Near the centre, the helium flux decreases inward and is zero at the centre, which is not shown in the Fig. 6.

Figs 7 and 8 show the largest growth rate of Eq. (18). The unseen parts of curves in Fig. 8 are as follows. The growth rates of models C, D and E are negative in the efficient convection zone inner than the vertical lines at about $M_r/M_\odot$ =19.5, 16 and 20, respectively. The curve of model C has a shape similar to those of models A and B in Fig. 7 though its upper part is not shown between $M_r/M_\odot = 20$ and 26.

The central convective cores of all four models are stable since the growth rates are negative. In model A the convection zone with large μ-gradient is unstable with the timescale of 10^4 sec. This growth rate of the instability decreases from model A to D. In model D, the intermediate insufficient convection zone is stable. The artificial increment in helium is as small as 10^{-4} in model E. In this model E, the outer part of the intermediate insufficient convection zone is unstable with the timescale of 10^8 sec. Since the artificial increment in helium is smaller in model E than in model D, the intermediate convection zone is probably unstable.

From Table 1 it is shown that in model D the convective element is larger than the radius of the stellar model by 4×10^4 times. Even in the stellar model C it is larger than the star radius by 4×10^2 times. To make the whole intermediate zone stable, the convective element must be much larger. Such extraordinarily large convective element never exists. Since the convective element cannot have enough adiabaticity to ensure the stability, the intermediate convective zone is unstable with the time-scale much shorter than the main sequence life time. The models with $\ell = \ell_{\rm b} \sim H_p$ cannot be calculated: when ℓ becomes longer and $\ell_{\rm b}$ becomes shorter, the radiative heat loss becomes significant and the μ-gradient becomes flatter: the μ-gradient is not large enough to keep helium flux positive in that case. We can, however, infer that the intermediate zone of such a model is unstable. Hence the convective zone with large μ- gradient will soon decay and cannot exist in the main-sequence stars even if it forms accidentally.

4. Concluding remarks

In this paper, the linear stability analysis of the inefficient convection zone with large μ-gradient is done with Eggleton's time-dependent equations of convection. If the radiative heat loss is artificially neglected, the convection zone with large μ-gradient is stable. If the convective element is larger, or if the mixing-length is smaller, the effect of the radiative heat loss becomes less important. However the necessary size of the convective element for the stability of the convective zone with large μ-gradient is larger than the stellar radius by several orders even if the mixing-length is much smaller than the pressure scale height. Hence the convection zone with large μ- gradient is never stabilized.

We briefly comment on the semiconvection problem. We consider the first massive star models by Sakashita & Hayashi(1959). These models consist of three zones, that is, the central convective core, the intermediate semiconvection zone and the radiative envelope. The intermediate zone of this model is static and is in accord with the Ledoux's neutral condition. As several authors argued, this intermediate semiconvection zone is unstable. Sakashita & Hayashi (1961) considered that the overshooting from the central convective core will modify the distribution of hydrogen in the intermediate zone. On the other hand Kato (1967) and Shibahashi & Osaki (1976) showed that the Ledoux intermediate zone is overstable. Gabriel (1970) proposed that the perturbation in hydrogen will sweep the intermediate zone and the Schwarzschild semiconvection zone will result. Thus the static Ledoux neutral convection zone is thought to be modified.

If the radiative heat loss from the convective element is negligible, the inefficient convection zone resembles the Ledoux neutral zone(Umezu & Nakakita 1988). The present work shows, however, that the radiative heat loss from the convective element cannot be negligible and that the inefficient convection is unstable thermally by itself, even if there is neither overshooting nor perturbation in chemical abundance. Hence the ineffective convection zone similar to the Ledoux neutral zone will decay rapidly even if it were formed accidentally. Moreover if the size of the convective element is nearly equal to the mixing-length as is ordinarily assumed, it seems that the μ-gradient in the ineffective convection zone cannot become large enough to keep helium flux positive. In addition, there are many problems in the stellar models constructed with the inefficient convection (Umezu 1989,1999), the inefficient convection zone does not seem to be an answer of the semiconvection problem.

Late in the main sequence evolution of the low-intermediate mass stars (about $1.5M_{\odot}$) convective core grows since CNO energy generation dominates the nuclear energy source. If main opacity is due to electron scattering, semiconvection will appear. Even in that case, the Ledoux semiconvection zone is probably unstable and is inappropriate.

The more elaborate theories of convection have been developed by many authors, for examples, Xiong (1981, 1986), Canuto (1999, 2000), Grossman et al. (1993) and Grossman & Taam (1996) using the one-point correlations of fluctuations. The results presented here should be examined from these new theories. Though the numerical simulation of semiconvection is very difficult (Merryfield 1995; Biello 2001; Paparella, Spiegel & Talon 2002; Meakin & Arnett 2007; Bascoul 2007), the development of computers will enable us to simulate numerically the instability of the convection zone with large μ-gradient.

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