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Solar differential rotation: origin, models and implications for dynamo

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Abstract. Helioseismology shows that the regions occupied by convection and differential rotation inside the sun almost coincide. This supports the leading theoretical concept for the origin of differential rotation as a result of interaction between convection and rotation. This talk outlines the current state of the differential rotation theory. Numerical models based on the theory reproduce the observed solar rotation quite closely. The models also compute meridional flow and predict that the flow at the bottom of the convection zone is not small compared to the surface. Theoretical predictions for stellar differential rotation as a function of the stellar mass and rotation rate are discussed and compared with observations. The implications of the differential rotation models for solar and stellar dynamos are briefly discussed.

Keywords : Sun: rotation – stars: rotation – dynamo

1. Introduction

The theory of global flows on the Sun is more fortunate in getting guidance from helioseismology than the twin-theory of global solar magnetic fields. Though the main concepts of the differential rotation theory were formulated before the emergence of helioseismology, the detailed helioseismological picture of the internal solar rotation helped to avoid spending time and efforts on studying theories not compatible with the picture. Now, about 150 years after the discovery of differential rotation of the sun by Carrington (1863), the origin of differential rotation seems to be well understood theoretically. Numerical models based on the theory reproduce solar rotation quite

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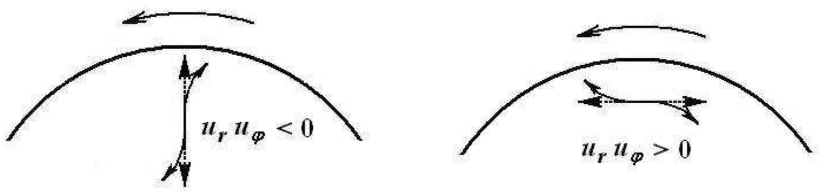

Figure 1. Illustration of angular momentum transport by turbulent mixing. Direction of rotation is shown on the top. *Left:* The original radial motion (dashed arrows) is disturbed by the Coriolis force so that the product $u_r u_\phi$ is negative and the angular momentum is transported downward. *Right:* $u_r u_\phi > 0$ for horizontal mixing producing upward transport of the angular momentum. Anisotropic mixing with different intensities of radial and horizontal motions is required for the net flux of the angular momentum to emerge.

closely (Tassoul & Tassoul 2004) and predictions for the differential rotation of stars are to some extent confirmed by observations.

This talk discusses the main processes thought of as responsible for the formation of solar differential rotation. Then, we shall see what the numerical models based on this theory produce for the sun and what they predict for solar-type stars. Implications of the results for solar and stellar dynamos are briefly discussed. The talk is mainly focused on the mean-field theory of differential rotation. The 3D numerical experiments were recently reviewed by Brun & Rempel (2009).

2. Theory

2.1 Differential rotation

Helioseismology shows that the decrease of angular velocity from the equator to poles seen on the surface of the sun survives throughout the convection zone up to its base but disappears shortly beneath the base (Wilson et al. 1997; Schou et al. 1998). The regions inside the sun occupied by differential rotation and convection almost coincide. This supports the theoretical concept that explains the differential rotation by the interaction between convection and rotation. Convective motions in rotating fluid are disturbed by the Coriolis force. The back reaction disturbs rotation making it not uniform.

Considering details of the process, it is easy to see that convective mixing along the radius tends to produce a state of sub-rotation with angular velocity increasing with depth. Fig. 1 shows that the fluid particles originally moving along the radius are deflected by the Coriolis force to attain azimuthal velocities. The product $u_r u_\phi$ is negative for both the upward and downward original motion; $\boldsymbol{u}$ is convective velocity, the standard spherical coordinates (r, θ, ϕ) are used. The negative value, $u_r u_\phi < 0$, means that the angular momentum is transported downward. This effect is so clear that almost every year brings new papers stating that it should be a general rule that angular velocity in mixed spherical bodies increases with depth.

The left panel of Fig. 1 does not show a complete picture, however. The right-hand panel shows that horizontal mixing tends to produce the state of super-rotation by transporting angular momentum upward. The cross-component, $Q_{r\phi}$, of the correlation tensor $Q_{ij} = \langle u_i u_j \rangle$ can be estimated as follows,

$$Q^{\Lambda}_{r\phi} = 2\tau\Omega \left(\langle u_\phi^2 \rangle - \langle u_r^2 \rangle \right) \sin\theta, \tag{1}$$

where Ω is angular velocity, and τ is convective turnover time, the meaning of the upper index Λ will be explained shortly. In order for the net (convective) flux of angular momentum to exist, intensities of radial and horizontal convective mixing should differ and the direction of the flux is defined by the sense of the anisotropy of the mixing (Lebedinskii 1941; Wasiutynski 1946; Biermann 1951).

The right part of Eq. (1) involves a key parameter of the differential rotation theory,

$$\Omega^* = 2\tau\Omega, \tag{2}$$

named the Coriolis number. The parameter measures the intensity of interaction between convection and rotation. The dramatic complication of the theory comes from the fact that the sun and absolute majority of cool stars have $\Omega^* > 1$ in the bulk of their convection zones. This means that the linear estimation (1) does not apply and the theory should be nonlinear in Ω^*. Nonlinear derivations of angular momentum fluxes show that for the nearly adiabatic stratification of convection zones the fluxes are parallel to the rotation axis and point to the equatorial plane in the rapid rotation case, $\Omega^* \gg 1$ (Kitchatinov & Rüdiger 1993). At small depths in the solar convection zone, the Coriolis number is smaller than one. The angular momentum fluxes have a radial inward direction in this case (Käpylä et al. 2004; Kitchatinov & Rüdiger 2005). As Ω^* increases with increasing depth in the convection zone, the fluxes change from an inward radial direction to an equatorward one parallel to the rotation axis.

The ability of convection to transport angular momentum even in the case of *rigid* rotation was named the Λ-effect (Rüdiger 1989). The rigidity of rotation is emphasized because turbulent viscosity can also transport angular momentum if rotation is not uniform. The Q^Λ of Eq. (1) is only a part of the total correlation of convective velocities, namely the part representing the Λ-effect. The total correlation includes the viscous part, Q^ν, also:

$$Q_{ij} = Q^{\Lambda}_{ij} + Q^{\nu}_{ij}, \quad Q^{\nu}_{ij} = -\mathcal{N}_{ijkl} \frac{\partial V_k}{\partial r_l}, \tag{3}$$

where $\boldsymbol{V}$ is the large-scale velocity and $\mathcal{N}_{ijkl}$ is the turbulent viscosity tensor. Viscous fluxes of angular momentum tend to reduce differential rotation. They increase with the inhomogeneity of angular velocity. Non-diffusive fluxes (the Λ-effect) depend on the angular velocity, not on its gradient. A steady state of differential rotation can 'to the first approximation' be understood as a balance between the Λ-effect and the eddy viscosity.

The approximation is, however, rather rough because it misses the important contribution to angular momentum transport made by the global meridional flow. The steady mean-field equation for the angular momentum balance reads (Kitchatinov 2005)

$$\mathrm{div}\left(\rho r \sin\theta \langle u_\phi \boldsymbol{u}\rangle + \rho r^2 \sin^2\theta\, \Omega \boldsymbol{V}^{\mathrm{m}}\right) = 0, \tag{4}$$

where $\boldsymbol{V}^{\mathrm{m}}$ is the meridional flow velocity. Substitution of an explicit expression for the correlations (3) of convective velocities into Eq. (4) gives an equation for the angular velocity. The equation is, however, not closed because it includes the yet undefinite meridional flow. The flow cannot be neglected or prescribed because meridional flow is produced by differential rotation (Kippenhahn 1963). Global flow in the convection zone of a star represents a self-regulating system: differential rotation produces meridional flow which in turn modifies the differential rotation.

2.2 Meridional flow

The origin of meridional flow is well illustrated by the (steady) equation for this flow,

$$\mathcal{D}(\boldsymbol{V}^{\mathrm{m}}) \;=\; \sin\theta\, r \frac{\partial \Omega^2}{\partial z} \;-\; \frac{g}{c_{\mathrm{p}} r}\frac{\partial S}{\partial \theta}, \tag{5}$$

where $z = r\cos\theta$ is the distance from the equatorial plane, S is the specific entropy, g is gravity, and c_{p} is the specific heat at constant pressure. The left part of this equation describes the viscous drag on the meridional flow (we will not need a rather bulky explicit expression for this term in the discussion to follow). The right part includes the two sources of the meridional flow.

The first term on the right of Eq. (5) represents centrifugal driving. If the angular velocity varies with z to decrease with distance to the equatorial plane, as it does in the sun, the centrifugal force produces a torque driving a flow to the poles near the surface and a flow to the equator near the bottom of the convection zone.

The second term on the right of Eq. (5) involves the so-called 'baroclinic driving' known also as the source of the 'thermal wind'. If polar regions are warmer then the equator, as they are on the sun (Rast et al. 2008), the baroclinic driving counteracts the centrifugal driving.

For solar and stellar conditions, each of the two terms on the right side of Eq. (5) is large compared to the left side. Therefore, the two terms nearly balance each other

- the condition known as the Taylor-Proudman balance. The balance is maintained mainly via the influence of meridional flow on the rotation law (Durney 1989).

Equations (4) and (5) again do not represent a closed system because the latitudinal entropy gradient is not defined.

2.3 Differential temperature

Turbulent heat transport in rotating convection zones is anisotropic. Not only does the heat transport coefficient depend on latitude (Weiss 1965), but the direction of the convective heat flux is not aligned with the entropy gradient (Rüdiger et al. 2005).

The convective heat flux,

$$F_i^{\mathrm{conv}} = -\rho T \chi_{ij} \frac{\partial S}{\partial r_j}, \tag{6}$$

depends on the structure of the thermal conductivity tensor,

$$\chi_{ij} = \chi_{\mathrm{T}} \left(\phi(\Omega^*)\delta_{ij} + \phi_{\|}(\Omega^*)\hat{\Omega}_i\hat{\Omega}_j \right), \tag{7}$$

$$\chi_{\mathrm{T}} = -\frac{\tau \ell^2 g}{12 c_{\mathrm{p}}} \frac{\partial S}{\partial r}, \tag{8}$$

where $\hat{\mathbf{\Omega}} = \mathbf{\Omega}/\Omega$ is unit vector along the rotation axis and the functions $\phi(\Omega^*)$, $\phi_{\|}(\Omega^*)$ of the Coriolis number (2) involve the rotationally induced anisotropy and quenching of the diffusivity (Kitchatinov et al. 1994). Even if entropy varies mainly with radius, the heat flux (6) deviates from the radial direction towards the poles. The poleward latitudinal heat flux is the main reason for the 'differential temperature' phenomenon in the mean-field theory. There were multiple attempts to measure differential temperature on the sun. Recent observations by Rast et al. (2008) suggest that the solar poles are warmer than the equator by about 2.5 K.

This small temperature difference between the equator and poles on the very hot sun has important hydrodynamical consequences. It can lead to differential rotation even without the Λ-effect (Durney & Roxburgh 1971): the differential temperature produces meridional flow by the baroclinic driving of Eq. (5) and the flow in turn transports angular momentum (4) to produce differential rotation. Models based on the anisotropic heat transport, however, do not reproduce the solar rotation. Nevertheless, only when differential temperature is accounted for can the helioseismologically detected internal rotation of the sun be reproduced (Kitchatinov & Rüdiger 1995; Miesch et al. 2006).

The angular momentum equation (4) together with the meridional flow equation (5) and entropy equation with the heat flux (6) represent a closed system that can

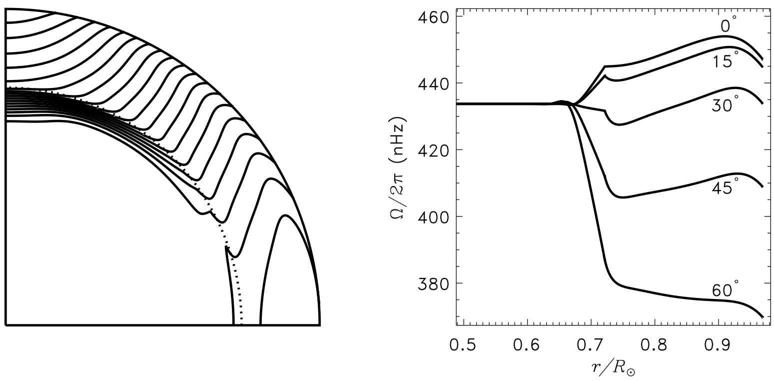

Figure 2. Angular velocity isolines (*left*) and depth profiles of the rotation rate for several latitudes (*right*) after the mean-field model of Kitchatinov & Olemskoy (2011).

be solved numerically for the distributions of angular velocity, meridional flow and entropy in a stellar convection zone. It should be noted that all that is needed for the numerical modeling - the Λ-effect, thermal conductivities and eddy viscosities - have been derived within the same approach. As a result, uncertainty in the model design was minimized and the mean-field models practically do not involve free parameters.

3. Models

3.1 The Sun

Fig. 2 shows the internal solar rotation computed with the mean-field model. The results are similar to the helioseismological rotation law.

The discussion in the preceding section refers to the convection zone only. Fig. 2 includes the tachocline region and the deeper radiation zone. The Figure was produced by joint use of two different models. Physical conditions in convection and radiation zones differ so much that it is not possible to cover both in one model. The rotation of the radiation core of Fig. 2 was computed with the magnetic tachocline model of Rüdiger & Kitchatinov (1997). The modeling of the tachocline does not influence the computation of the differential rotation of the convection zone in any way but just uses the results of this computation as a boundary condition. There is no space for discussing tachocline physics here. We just mention that up to now it has been possible to explain simultaneously the uniform rotation in the deep radiation core and a slender tachocline on its top only by an effect of an internal relic magnetic field (Charbonneau & MacGregor 1993; Rüdiger & Kitchatinov 1997; MacGregor & Charbonneau 1999; Denissenkov 2010). The tachocline of Fig 2 is produced by a weak internal poloidal field of about 10^{-2} Gauss.

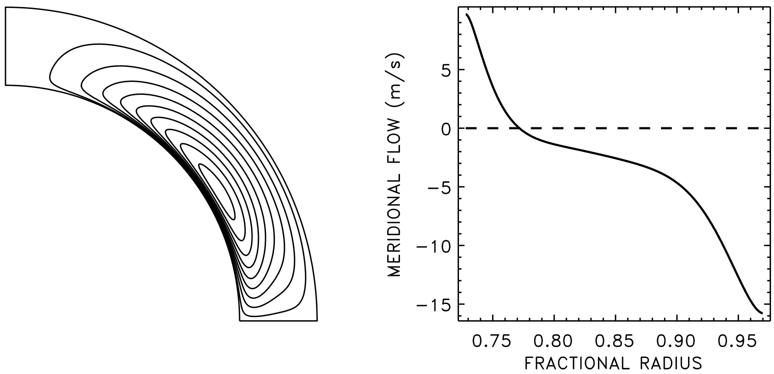

Figure 3. Meridional flow after the same model as Fig. 2. *Left*: Meridional flow stream-lines. *Right:* Radial profile of meridional velocity for 45° latitude. Negative velocity means poleward flow.

The angular velocity distributions in theoretical models are symmetric about the equator and regular near the poles. This implies that the angular velocity isolines are normal to both the equatorial plane and the rotation axis. The isorotational surfaces, cylinder-shaped near the equator and disk-shaped near the poles, are, therefore, elementary consequences of the global symmetry of the problem (for alternative opinion see Balbus 2009). The nearly radial isolines of Fig. 2 at middle latitudes are, however, not trivial. They are the consequence of the Taylor-Proudman balance in the bulk of the convection zone (see Section 2.2 above). Only with allowance for the effect of differential temperature can deviations from cylinder-shaped isorotation surfaces be reproduced.

Differential rotation models compute also the distributions of entropy and the meridional flow. The flow is increasingly recognized as important for the solar dynamo (Choudhuri 2008). The flow at small depths can be probed by helioseismology (Zhao & Kosovichev 2004). Theoretical modeling remains, however, the only source of knowledge on the deep meridional flow. The typical structure of the simulated circulation is shown in Fig. 3. The plot shows the flow distribution up to the base of the convection zone. Beneath the base, the meridional velocity rapidly decreases with depth (Gilman & Miesch 2004).

Meridional flow results from a (small) disbalance between two terms in the right side of Eq. (5). Observations of the flow (Komm et al. 1993) indicate that some deviations from the Taylor-Proudman balance are present in the sun. The flow of Fig. 3 is relatively small in the bulk of the convection zone and increases towards the zone boundaries. Such a structure of the flow is related to the boundary layers (Durney 1989). The Taylor-Proudman balance is not compatible with stress-free boundary conditions. As a result, boundary layers form where the balance is violated and the sources of meridional flow are relatively large. The bottom flow of Fig. 3 is faster than

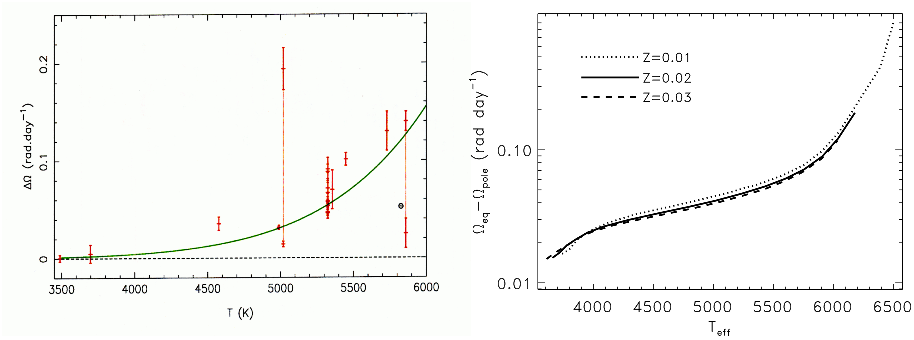

Figure 4. *Left:* Dependence of surface differential rotation on effective temperature for rapidly rotating solar analogues observed using the Doppler imaging techniques (Barnes et al.2005; figure courtesy of John Barnes). *Right:* Same dependence computed with the differential rotation model (Kitchatinov & Olemskoy 2011). Lines of different styles correspond to different metallicities Z.

usually assumed in advection-dominated dynamo models. The first dynamo-model with a fast near-bottom flow was recently produced by Pipin & Kosovichev (2011).

Another implication for dynamo models is related to the value of the magnetic eddy diffusivity. The diffusivities in the differential rotation models are not prescribed but expressed in terms of the entropy gradient, as it is done in Eq. (8) for turbulent thermal diffusion. Characteristic values of the resulting turbulent viscosities and diffusivities are about 10^{13} cm^{2}s^{-1} for the sun; global circulation models with smaller diffusivities are unstable (Tuominen et al. 1994). Theories of turbulent transport coefficients and 3D simulations (Yousef et al. 2003) both suggest that the magnetic Prandtl number is of the order of one, i.e., the eddy magnetic diffusivity is also about 10^{13} cm^{2}s^{-1}. This value is larger than usually assumed in solar dynamo models.

3.2 Cool and Solar-Type Stars

The mean-field models for differential rotation can, of course, be applied to convective stars other than the sun. The models, which do not include the effects of magnetic fields, always predict solar-type rotation with the equator rotating faster than the poles. They also predict that dependence on rotation rate for a star of given structure is relatively small. The main prediction, however, is that the surface differential rotation increases with stellar mass (Kitchatinov & Rüdiger 1999).

The predictions are in at least qualitative agreement with observations. Differential rotation of two solar twins rotating about three times faster than the sun was measured recently using high precision photometry of the *MOST*-mission (Croll et al. 2006; Walker et al. 2007). In both cases, the amount of the surface differential rotation

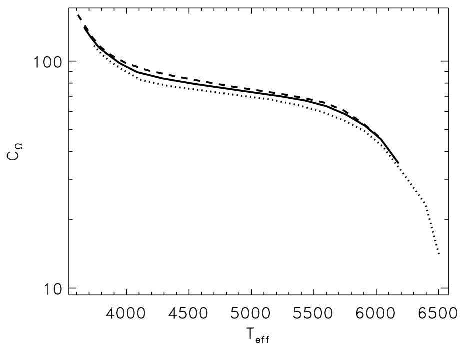

Figure 5. C_Ω dynamo number (9) as the function of stellar surface temperature after the same model as the right panel of Fig. 4. Different line styles show the results for different chemical compositions.

was close to solar value. Differential rotation measurements by Doppler imaging for young rapidly rotating stars were summarized by Barnes et al. (2005). Fig. 4 compares the dependence on stellar surface temperature they found with computations of Kitchatinov & Olemskoy (2011).

Both plots of Fig. 4 suggest that the hottest convective stars possess the largest differential rotation. As the rotational shear is important for dynamos, the question arises whether the strong differential rotation of F-stars implies over-normal dynamo activity? Surprisingly, the answer is negative. Dynamo theory estimates the efficiency of differential rotation in generating magnetic fields by the variety of the magnetic Reynolds number conventionally notated as C_Ω,

$$C_\Omega = \frac{\Delta\Omega H^2}{\eta_{\rm T}}, \tag{9}$$

where $\Delta\Omega$ is the angular velocity variation within the convection zone, H is the convection zone thickness, and $\eta_{\rm T}$ is the turbulent magnetic diffusivity. Fig. 5 shows the dependence of C_Ω on stellar effective temperature after the same model as the differential rotation plot of Fig. 4. The two Figures display, however, opposite trends. The C_Ω-parameter decreases with temperature. The large differential rotation of F-stars is much less efficient at producing toroidal magnetic fields than the almost uniform rotation of M-stars. This is in agreement with the old idea of Durney & Latour (1978) that convective dynamos cease to operate at about spectral type F6.

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