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Photometric parameters, distance and period-colour study of contact binary stars in the globular cluster ω Centauri

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Abstract. We present V passband photometric solutions of six contact binaries in the globular cluster ω Centauri using the Wilson-Devinney code. We found that four of the systems are H subtype W UMa binary systems and remaining two are W-subtype W UMa systems. Four of the systems show the O'Connell effect in their light curves and brightness at 0.75 phase is about δV=0.04$-$0.05 less than at 0.25 phase. We show that one dark/cool spot solution on secondary companion satisfactorily fits the observed light curves for the respective contact binaries. The derived spot values are not unique and they are taken to be tentative at best. The colour index of the six contact binaries were used to derive the distance modulus of ω Centauri and found an average value of $\sim$13.60 which is close to the observed value of 13.97. We also derive the distance to each of the studied variables. The period-colour relation of the contact binaries of ω Centauri is compared with the period-colour relation of Hipparcos data (field W UMa type) and contact binaries present in other globular clusters.

Keywords : stars: binaries (including multiple): close – stars: binaries: eclipsing – stars: binaries: general – stars: individual: W UMa systems

1. Introduction

The most massive Galactic globular cluster ω Centauri (NGC 5139) is one of the highly studied systems from ground-based telescopes (Villanova et al. 2007) as well as from the Hubble Space Telescope (Ferraro et al. 2004). The central position of ω Centauri is α_{2000}=13^{h}26^{m}47.24^s, δ_{2000}=-47^{o}28$'$46$''$ (Anderson & van der Marel 2010) with an age spread of 2$-$4 Gyr within the

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cluster (Stanford et al. 2006). Multi-wavelength studies show that the cluster hosts different stellar populations and at least four peaks in the metallicity distribution function were observed ranging from [Fe/H]= -1.75 to -0.75 (Johnson et al. 2009). Perhaps the most important puzzling discovery of the system came from Piotto et al. (2005) who showed that the bluer main sequence (bMS) component is relatively metal rich compared with the contemporary red main sequence component (rMS). Despite its huge mass, $2-7\times10^6$ $M_\odot$ (van de Ven et al. 2006 and references there in), the marked observational lack of deep potential is still a mystery (Gnedin et al. 2002). The distance is about $\sim$4.8 kpc with a distance modulus of 13.75 mag. This cluster is considered to be formed due to the gravitational interaction between the Milky Way and a dwarf spheroidal galaxy (Bekki & Norris 2006).

Many optical surveys were carried out in order to know the population of variable stars in this cluster. The first massive survey was conducted by the Optical Gravitational Lensing Experiment (OGLE) I team (Kaluzny et al. 1997) and later a more elaborate search was carried out under the project Cluster AgeS Experiment (CASE) by Kaluzny et al. (2004). The latter survey revealed nearly 117 new variables in which 26 were eclipsing binaries. A larger area of this cluster was probed up to 0.75 deg^2 in which 58 eclipsing binaries were discovered (Weldrake, Sackett & Bridges 2007). Similar kinds of observations were carried out by Bellini et al. (2009) to study the proper motion of the stars.

Since detailed photometric analysis has not been carried out for the eclipsing binaries of W UMa type (EW) present in this cluster, we have analyzed the light curves of six eclipsing binary systems V206, V207, V208, V214, V236 and V240 which have relatively less scatter in V passband light curves.[1] The remaining eclipsing binaries of EW type have comparatively large scatter and hence were not used for this analysis. Similar kinds of studies have been carried out on the W UMa type binaries in the open clusters NGC 6791, Be 33, Be 39 and NGC 7789 (Rukmini & Vivekananda Rao 2002; Rukmini, Vivekananda Rao & Sriram 2005; Sriram, Ravi Kiron & Vivekananda Rao 2009; Sriram & Vivekananda Rao 2010). Such studies are important as they give information on radii and temperature of binary components which are further used to determine the distance of respective systems and the host (cluster) (e.g. Deb & Singh 2011). It was found that in four binaries out of six, the O'Connell effect was clearly observed. The V passband light curve at 0.75 phase have statistically less brightness when compared to 0.25 phase brightness ($\delta V_{phase(0.25-0.75)}$=0.04$-$0.05). The aim of the present paper is to derive the photometric elements of the selected W UMa type systems discovered in ω Centauri cluster using the W-D Code and put constraints on the mass ratio for individual systems.

2. Data and analysis

ω Centauri was observed during 1999 February 6$-$7 to 2000 August 9$-$10 using CCD photometry on the 1.0-m Swope telescope at Las Campanas Observatory by the Kaluzny group (Kaluzny

[1]The data for the respective stars were obtained from the URL: http://case.camk.edu.pl/results/oCEN/tab.html and it can also be obtained from Vizier link.

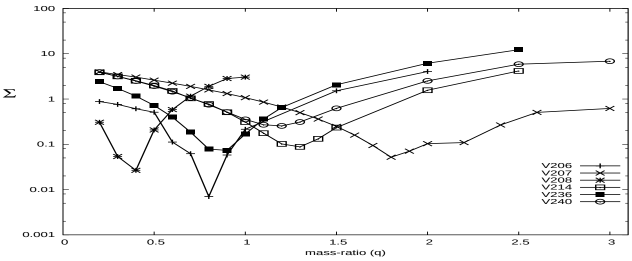

Figure 1. q vs. Σ of the respective variables. The y-axis is plotted in log scale to have clarity.

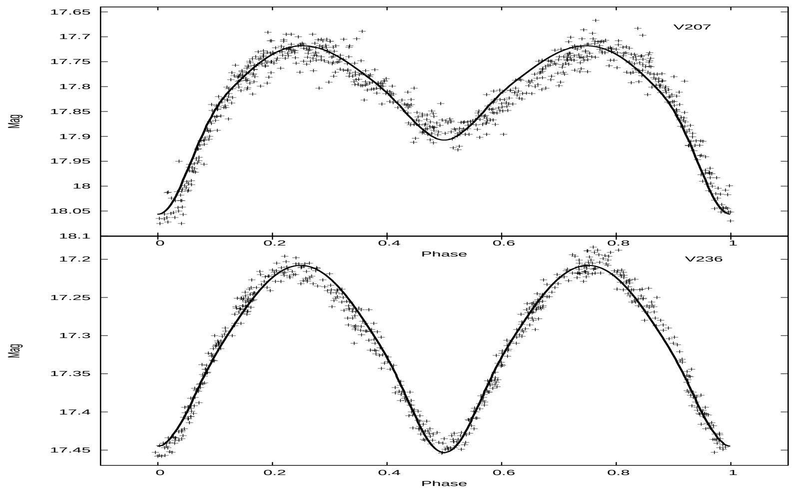

Figure 2. The best fits to the V passband light curves of the two variables. The lines represent the best fits to the observed data.

et al. 2004) under the project CASE. The collected data during 49 nights revealed a total of 117 variables of which 35 were SX Phe variables, 26 were eclipsing binaries and 17 were RR Lyrae-type stars. We have chosen six EW type variables (V206, V207, V208, V214, V236 and V240) whose light curves resemble W UMa type variability in order to derive the photometric parameters using the W–D code. The other EW type variables' light curves were found to be more scattered and hence not considered for the W–D analysis. The V passband data were taken from Kaluzny et al. (2004; *VizieR Result page*).

3. Photometric solutions

The photometric solutions were obtained by using the 2003 version of the Wilson-Devinney Code (van Hamme & Wilson 2003) with an option of non-linear limb darkening via a square root law along with many other features (Wilson & Devinney 1971; van Hamme & Wilson 2003). We followed the procedure as discussed in our earlier work (Sriram & Vivekananda Rao 2010). Starting with the assumption that the systems are detached (Mode 2), the differential corrections always converged to the contact mode (Mode 3) solution. In this mode, the dimensionless sur-

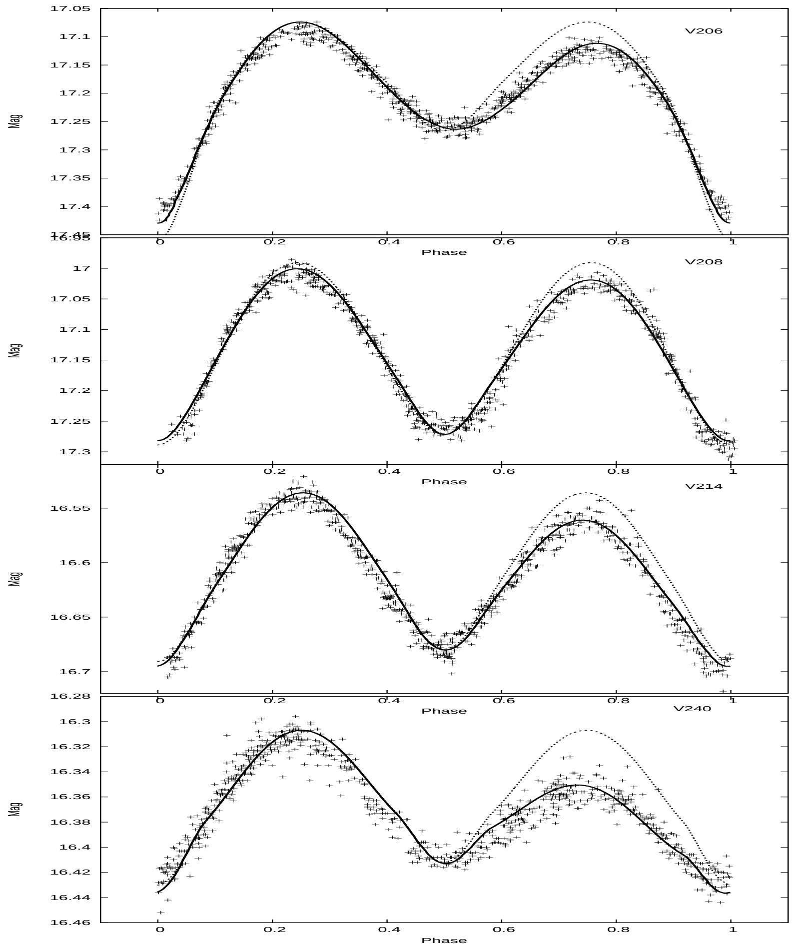

Figure 3. The best fits to the V passband light curves of the four variables. Thick lines show the best fits of spot solutions and dashed lines show the fits without spot solutions. The difference in the un-spotted and spotted solutions can be seen at 0.75 phase

face potentials Ω of both the components are equal but can have thermal differences between the components (Kopal 1959; Wilson 1979). The following parameters were set fixed during the fitting procedure. The effective temperature for the primary star, which is taken on the basis of un-reddened B-V values using Bellini et al. (2009) and Allen's table (2000). Initial value of secondary component effective temperature ($T_{e,c}$) was assumed to be equal to primary component's temperature. The values of limb darkening coefficients, x_h and x_c = 0.68 (Diaz-Cordoves, Claret & Gimenez 1995) for V band. The gravity-darkening coefficients g_h and g_c for V passband was set to 0.32 (Lucy 1967) and the values of bolometric albedo A_h and A_c were fixed at 0.5 which is appropriate for the convective nature of the component stars (Rucinski 1969).

To determine the most important parameter, the mass ratio q, the mean temperature of cool star T_c, the surface potentials (Ω_h=Ω_c), monochromatic luminosity L_h and inclination i were varied for a fixed value of the mass ratio. To determine an accurate value of the mass ratio, the respective parameter was set free along with other adjustable parameters until a convergent solution was arrived at on the basis of the minimum value of ΣW(O-C)^2 (Fig. 1). It was found that in four variable systems, the O'Connell effect is clearly observed and most probably is due to the spot activity on the cool star (Fig. 3). It is found that at 0.75 phase, brightness is decreased by 0.02 to 0.04 when compared to brightness at 0.25 phase. Due to this observed change in the level of brightness at 0.25 and 0.75 phase, we have added a cool/dark spot on the secondary star to obtain the spot parameters along with the other photometric parameters. For this purpose, we have taken un-spotted solution for each variable and carried out the fitting procedure. We found that ΣW(O-C)^2 decreases which indicates that the spot solution is required for the respective variables (Table 1). The theoretical light curves were obtained using the LC program of Wilson-Devinney and the fits are shown in Figs. 2 and 3.

4. Discussions and results

We have analyzed six eclipsing variables present in the ω Centauri cluster. We used the W-D Code to derive the photometric solutions for each system. We found that for four variables V206, V208, V214 and V240, the theoretical light curves were not fitting the observations without the spot. We had to incorporate the spot to get the best fits to the observed light curves of these variables. The Σ_{spot} value decreased compared to the $\Sigma_{un-spotted}$ one (Table 1) which clearly indicates that the presence of the spot modifies the theoretical light curves and match with the observed V passband light curves. However, the derived spot values are not unique and they are taken to be tentative at best. The results on each of the systems are discussed below.

4.1 V206

The proper sinusoidal variation of the light curve indicates the system to be of W UMa type. The period of the system is around $\sim0^d.307351$ and colour is B$-$V=0.290 (Bellini et al. 2009). The V passband amplitude is different at 0.0 and 0.50 phase which clearly indicates the temperature difference between primary and secondary components (Fig. 3). Our solution suggests a temperature difference of δT=2630 K. This source shows a clear O'Connell effect due to a difference in brightness level at phase 0.25 and 0.75. A dark spot of 16° radius close to the equator is needed to get the best fit to the light curve (Table 1). The spot temperature is not much different from the temperature of the photosphere, T_s/T_*=0.90. We found that the mass ratio is greater than 0.72 which makes this system fall in H-subtype W UMa type system (Csizmadai & Klagyivik 2004).

4.2 V207

The period of this system is $0^d.275867$ and the colour is B$-$V=0.394. The light curve maxima

Table 1. The photometric elements obtained for the six variables by using the W-D method.

Element		V206	V207	V208	V214	V236	V240
Period (days)		0.307351	0.275867	0.305526	0.341804	0.296318	0.331884
$T_{e,h}{}^{a}$ K		7550	6786	7610	7630	6123	7379
$T_{e,c}K$		4920±45	5242±57	7619±36	7791±68	6231±30	6871±87
q		1.104±0.0024	1.78±0.05	0.47± 0.02	1.27±0.12	0.87±0.06	1.16±0.02
i°		73.24±0.20	70.68±0.38	71.32±0.54	73.07±0.78	71.23±0.53	70.48±0.64
Ω		3.9113±0.0052	4.9553±0.0131	2.6971±0.0035	4.1783±0.0208	3.5180±0.0141	4.0852± 0.0303
r_h	pole	0.3486±0.0049	0.3080±0.0012	0.4410±0.0073	0.3313±0.0025	0.3671±0.0020	0.3231±0.0035
	point	0.3548±0.0057	0.3143±0.0014	0.4618±0.0092	0.4390±0.0216	0.4947±0.029	0.3888± 0.0106
	side	0.3658±0.0060	0.3217±0.0015	0.4740±0.0101	0.3466±0.0032	0.3861±0.0026	0.3361±0.0042
	back	0.3975±0.0084	0.3541±0.0023	0.5091±0.0147	0.3771±0.0047	0.4162± 0.0038	0.3598±0.0059
r_c	pole	0.3651±0.0078	0.4054±0.0012	0.3101±0.0106	0.3766±0.0024	0.3435±0.0021	0.3542±0.0034
	point	0.3728±0.0046	0.4128±0.0013	0.3198±0.0122	0.4947±0.0021	0.4656±0.0308	0.4262±0.0103
	side	0.3841±0.0085	0.4297±0.0015	0.3266±0.0134	0.3966±0.0030	0.3601±0.0026	0.3701±0.0041
	back	0.4150±0.0050	0.4582±0.0020	0.3744±0.0270	0.4255±0.0042	0.3910±0.0039	0.3935±0.0055
$L_h{}^{b}$		0.8151±0.0122	0.6453±0.0087	0.4851±0.0110	0.4721±0.0058	0.4931± 0.0078	0.5623±0.0067
L_c^a		0.1848	0.3546	0.5149	0.5278	0.5068	0.4376
L_3		0.0	0.0	0.0	0.0		
x_h		0.68	0.68	0.68	0.68	0.68	0.68
x_c		0.68	0.68	0.68	0.68	0.68	0.68
Σ		0.0068	0.0130	0.0146	0.0054	0.0033	0.0067
A_h^a		0.5	0.5	0.5	0.5	0.5	0.5
A_c^a		0.5	0.5	0.5	0.5	0.5	0.5
Spot Parameters							
(Co-lattitude)ϕ°		78.53±6.0		72.83±4.82	77.35±1.34		88.23±1.37
(Longitude)θ°		255.67±4.7843		210.58±5.8	235.89±2.52		256.807±2.28
(Angluar Radius)γ°		15.82±3.0		20.88±1.071	16.157±0.45		19.65±0.68
T_s/T_*		0.91±0.01		0.93±0.01	0.79±0.02		0.73±0.03
Σ_{spot}		0.0031		0.0073	0.0036		0.0019

[a] Fixed parameters; [b] In units of total system at phase 0.25

and minima are not equal and the computed effective temperatures of the primary and secondary components show a difference of δT$\sim$1545 K (Fig. 2, Table 1). The best combination of q and i is $\sim$1.78 and $\sim$71° respectively. The inverse mass ratio value q_{inv}=0.56, a typical value found in W-subtype W UMa systems. The light curves do not show any signature of the O'Connell effect.

4.3 V208

The light curve shows clear signature of the O'Connell effect which can be observed in the un-spotted and spotted solution fits (Fig. 3). The period of the binary system is $\sim 0^d.305526$ and the value of B$-$V=0.247. The best combination of q and i was found to be $\sim$0.47 and $\sim$69° respectively. The spot with 21° gave the best fit which can be seen from the fits of the un-spotted and spotted solutions of the respective system light curve.

4.4 V214

In this case the spot temperature is 20% less than the photosphere temperature, T_s/T_*=0.79. A dark spot close to equator ($\sim$+12 degree) and of 16° radius matches well the observed light curve. The computed mass ratio was found to be q_{inv}= 0.79 and falls into the category of H-subtype W UMa system. The period of the variable was found to be $\sim 0.^d341804$ and value of B$-$V=0.239.

4.5 V236

The period of the systems is $0^d.296318$ and B$-$V=0.540. There is a small difference in the primary and secondary component temperature, δT, of about 100 K. The light curve shows no signature of the O'Connell effect. The mass ratios and inclinations were found to be around q$\sim$0.87 and i$\sim$71°. The derived parameter values suggest that the system is of W-subtype W UMa type variable.

4.6 V240

The un-spotted and spotted solution fits favour the latter solution. The brightness magnitude at 0.75 phase is $\sim$0.05 factor less than brightness magnitude at 0.25 (Fig. 3). A dark spot on the secondary component near the equator with a radius of $\sim$20° gives a good fit to the observed light curve (Table 1). The period of the system is $0^d.331884$ and B$-$V=0.289.

We found that among the studied variables, three variables (V206, V207, V240) have significant temperature difference between the primary and secondary components (Table 1). The high temperature difference is often observed in EB-type systems (Martignoni et al. 2009) and are important from the point of view of the thermal relaxation oscillation (TRO) theory (Flannery 1976; Lucy 1976; Robertson & Eggleton 1977; Li, Han & Zhang 2004a, b, 2005). During

Table 2. Distance estimates of the six variables.

Variable	m-M	Distance (pc)
V206	13.86	5942.92
V207	13.97	6223.00
V208	13.87	5942.92
V214	13.65	5370.31
V236	13.10	4168.69
V240	13.13	4226.66

the thermal relaxation oscillation there exists a phase where the degree of thermal contact is low and the system exhibits EB-type variability in the light curves. The high photometric mass-ratio values for V206 (q_{inv}=1/q=0.90) and V240 (q_{inv}=0.86) suggest that the respective variables are at the beginning of the contact phase because of high mass ratio and V207 ($q_{inv}\sim$0.56) is at the end of the contact phase. These kinds contact binaries (EB-type) are important to constrain the theoretical models about the structure and evolution of W UMa-type systems.

However, since these are partial eclipsing over contact binaries, the solutions derived by us are only preliminary. As the mass ratios are the most important parameters in the light curve modeling, we need spectroscopic radial velocity measurements to ascertain the values of q (Rucinski 2001).

5. Distance determination of the six variables

Distance is an important parameter for any star/variable. Hence we have calculated the distances of the above studied six variables. Rucinksi & Duerbeck (1997) derived a calibrated relation, $M_v^{BV} = -4.44 \log P + 3.02(B\text{-}V)_o + 0.12$ using the Hipparcos data. Using this relation, we obtained the value of M_v for the six variables and then calculated the respective distance modulus assuming interstellar absorption A_v=0.33 (where $A_v\sim$3 E(B-V), E(B-V)=0.11). Table 2 shows the distance modulus for each variable along with its distance. We obtain the average distance modulus of about (m-M)$\sim$13.60 which is close to the value of 13.97 obtained from other studies (Harris 1996). Bellini et al. (2009) found that all the six studied variables belong to the cluster with membership probability $\sim$99%. We independently show that the average distance modulus value derived suggests that these variables belong to the host cluster.

6. Period-colour relation

We studied the period vs intrinsic colour $(B\text{-}V)_o$ and $(V\text{-}I)_o$ for eclipsing binaries (EW type) in ω Centauri. The B-V values for contact binaries (studied above) along with other W UMa type systems were taken from Kaluzny et al. (2004), and corresponding V-I values were taken

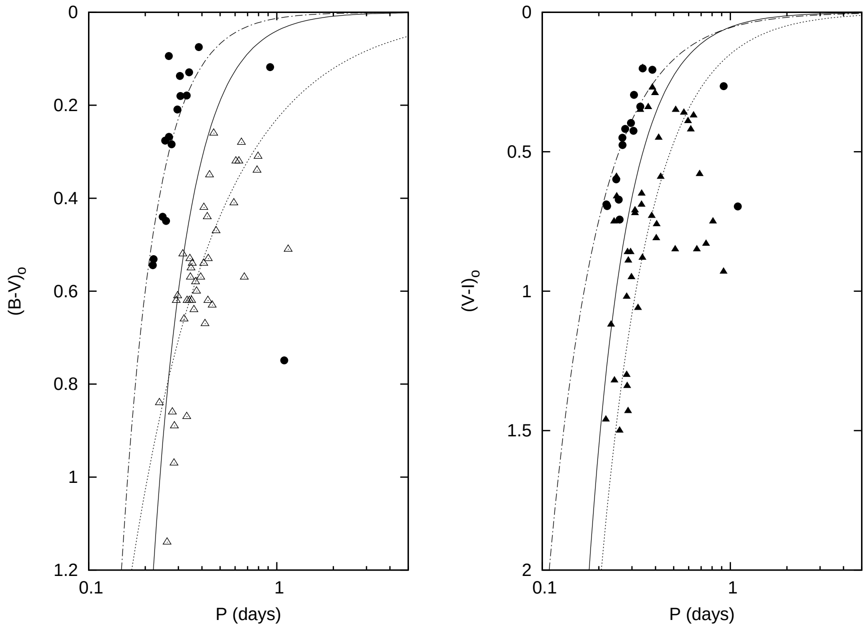

Figure 4. Period vs colour (B-V)$_o$ (left panel), (V-I)$_o$ (right panel) diagrams for the contact binaries are shown. The filled circle symbols correspond to contact binaries in the globular cluster Omega Centauri and dot-dash lines show the least-square fits (in both the panels). Open triangle symbols corresponds to Hipparcos data of field EW systems and dotted line show the respective fit (left panel). Filled triangle represents EW systems from the work of Weldrake's group (see text) and the corresponding fit is shown with dotted lines (right panel). The thick lines in both the panels show the period-colour relation fit as obtained by Rucinski (2000) by studying contact binaries in other globular clusters (see text).

from Bellini et al. (2009). The intrinsic colours were obtained by assuming reddening values E(B-V)$\sim$0.11 (Lub 2002) and E(V-I)$\sim$0.14. As no systematic value of E(V-I) was found for ω Centauri by any author, we adopted the following procedure for deriving E(V-I). We have used the tabulated values from Schlegel, Finkbeiner & Davis (1998) to calculate the extinction.

$$\begin{aligned} E(V-I) &= A_V - A_I \\ &= \left(\frac{A_V}{E(B-V)} - \frac{A_I}{E(B-V)}\right) \times E(B-V) \\ &= (3.315 - 1.940) \times 0.11 \\ &= 0.151 \end{aligned}$$

There was a study by Dean, Warren & Cousins (1978) wherein the value Y=1.25$\pm$0.01 (Y=E(V-I)/E(B-V)) was obtained using Cephied variables. Similar value (Y$\sim$1.24) was independently obtained and was used to derive the true colour by Rucinski (2000). These results indicate a value of E(V-I)$\sim$0.137. We have used a value of E(V-I)=0.144 (average of the values from Schlegel et al. (1998), the one derived here (0.151), and from Dean et al. (1978) and Rucinski (2000) of $\sim$0.137) to obtain the true colour (V-I)$_o$ for the respective contact binaries in ω Centauri.

Fig. 4 (left panel) shows the period vs true colour, (B-V)$_o$ (marked with filled circle) and is compared with the period-colour relationship for the field W UMa type variables (data are taken from Rucinski & Duerbeck 1997). The contact binary systems in ω Centauri satisfy a relation (B-V)$_o$ = 0.013 P(d)$^{-2.38}$ which is different from the field stars' (marked with open triangles) period-colour relation ((B-V)$_o$= 0.23 P(d)$^{-0.93}$). Similar kind of relation ((B-V)$_o$ = 0.04 P(d)$^{-2.25}$) was obtained by Rucinski (2000) by studying contact binaries present in other globular clusters. The right panel of Fig. 4 shows a similar plot using (V-I)$_o$ values. We have also used V–I values of EW discovered in ω Centauri by Weldrake's group (Weldrake, Sackett & Bridges 2007). Few of the EWs discovered by Weldrake's group matches with those discovered by Kaluzny et al. (2004). Weldrake, Sackett & Bridges (2007) suggested that most of EBs discovered belong to the parent cluster. The Kaluzny data (marked with filled circles) satisfy a relation (V-I)$_o$= 0.055 P(d)$^{-1.62}$ and Weldrake's data (marked with filled triangles) satisfy a relation (V-I)$_o$= 0.15 P(d)$^{-1.64}$. For other globular clusters' contact binaries, the respective relationship was found to be (V-I)$_o$= 0.053 P(d)$^{-2.1}$ (Rucinski 2000).

The fits of period vs (B-V)$_o$ between EW variables in ω Centauri and EW field stars (Hipparcos data) show a difference and it could be due to uncertainty in the B$-$V values, as period is known to a high degree of accuracy. A low value of metallicity in the respective contact binary variables can also account for the shift from the EW field star's locus and it should show large effect on the observed period than in the colour index (Rucinski 2000).

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