Some properties of merger remnants of interacting galaxies modeled as $n = 4$ polytrope

P. M. S. Namboodiri

Indian Institute of Astrophysics, Bangalore 560034, India

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Abstract. Galaxies interact in a multitude of ways with their environment. Such interactions can alter the morphological type of galaxies, trigger star formation and even produce active galactic nuclei. One of the important parameter in a galaxy collision is the impact parameter. Numerical simulations have been performed to study the effect of changing the impact parameter in a galactic collision. The initial density distribution in the galaxy corresponds that of a polytrope of index $n = 4$. The galaxy model does not include the dark matter halo component and therefore is not expected to mimic all properties of a real galaxy. Both merging and non-merging collisions of equal mass galaxies have been studied to see where the transition occurs between these two processes. Merging occurs when the closest approach distance of the galaxies is less than three times its half-mass radius. The merging time increases with the distance of closest approach. The density profiles of the merger remnants follow a $r^{1/4}$ law in the inner parts and deviate from it in the outer regions. Distant encounters do not result in merging and the galaxies remain almost intact with only negligible change in its mass and internal energy.

Keywords: galaxies – dynamics – collisions – numerical simulations

1. Introduction

The environment plays an important role in shaping the structure of a galaxy. Collisions of galaxies of comparable mass that are widely separated do not coalesce in a Hubble time. On the other hand galaxies with close companions are expected to merge in a few crossing times. It is being realized that many elliptical galaxies have been formed by the
merger of spirals. Giant luminous galaxies at the cores of dense clusters are supposed to have been formed by the merger of smaller companions. Massive elliptical galaxies are expected to have been formed in mergers between bulge-dominated gas-poor (dry) early type galaxies (Khochfar & Burkert 2003; Khochfar & Burkert 2005). The properties of the remnants of gas-free mergers of spheroids are in general similar to those of observed elliptical galaxies (Naab & Burkert 2000; Gonzalez-Gracia & van Albada 2005). In a pair of colliding galaxies, moving in an unbound orbit, $E \geq 0$ before the encounter where $E$ is the energy due to the orbital motion of the two galaxies. If after the encounter $E < 0$, then the two galaxies will form a double system. Hence the condition for merging may be stated as $|\Delta E|/E \geq 1$ where $\Delta E$ is the change in the orbital energy during the encounter. On the other hand if $U$ is the initial internal energy of a galaxy and $\Delta U$, its change during an encounter, then $\Delta U/|U| \geq 1$ implies that the galaxy will suffer considerable disruption during the encounter (Alladin & Narasimhan 1982). Merging and disruption are two important processes in the dynamical evolution of a binary stellar system. The ratio of the times of disruption and merging is given by

$$\frac{t_d}{t_m} \approx \frac{6}{(5 - n)} \frac{a M}{R M_1}$$

(1)

where $t_d$ and $t_m$ are the disruption and merging times, $a$ is the orbital radius, $R$ is the radius of the galaxy with mass $M$, $M_1$ is the mass of the perturber and $n$ is the polytropic index describing the density distribution of the galaxy (Alladin & Parthasarathy 1978). It can be seen that if the galaxies are centrally concentrated (i.e., $n = 4$) and have similar masses, merging occurs more rapidly than disruption. On the other hand, if the masses are dissimilar, the interaction between them is likely to cause considerable disruption to the less massive companion and in this case the disruption time could be shorter than the merging time.

Elliptical galaxies constitute roughly 10 per cent of the bright galaxies in the general field (Fall 1981). They are almost gas-free systems. Bound elliptical pairs tend to have similar mass and their separation and velocity difference are small compared to other types of galaxies (Charlton, Whitmore & Gilmore 1995). Most of the earlier simulations mainly concentrated on the merging aspect of spherical galaxies having different masses and uniform density distribution (White 1978; Roos & Norman 1979; Dekel et al. 1980; Farouki et al. 1983; Borne 1984; Aguilar & White 1985; Navarro 1989, 1990; Namboodiri & Kochhar 1990; Gonzales-Gracia & van Albada 2005 etc). Despite the fact that spiral galaxies merge to form elliptical galaxies, such remnants are considered to be low mass ellipticals formed in minor mergers (i.e., $M_1/M_2 > 3.5$; $M_1$, $M_2$ being the mass of the galaxies). More than 50 per cent of present-day elliptical galaxies brighter than $M_B \sim -18$ in clusters have experienced major mergers that were not between two classical spiral galaxies (Khochfar & Burkert 2003). The present work investigates both merging and non-merging collisions of equal mass gas-free galaxies by varying the pericentre distance of the orbit of the colliding galaxies. The density distribution of the galaxies is represented by that of a polytrope of index $n = 4$. The choice of $n$ is appropriate because the density distributions of ellipticals and bulges of disk galaxies are generally assumed to follow the
above density profile. The values used for the impact parameter cover both merging and non-merging types of collisions. The simulations are expected to give an idea about the transition between these two processes. They are also used to study the development of the density profiles in the merger remnants. The numerical model of the present work is described in Section 2. The results are presented and discussed in Section 3 and the conclusions are given in Section 4.

2. Initial conditions

A pair of equal mass galaxies with mass M is considered. The galaxy is modeled as a spherical non-rotating cluster of identical point particles. The positions and velocities of the particles are generated using random numbers so that the resulting density distribution closely follows a $n = 4$ polytropic model and it is considered to be the appropriate density profile of a typical elliptical galaxy. The simulations use a limited set of values for the impact parameter and the influence of massive halos are not included. The models presented are purely dissipationless taking into account the stellar component of a galaxy. According to Kormendy & Bender (1996), dissipation becomes less important for the formation of massive elliptical galaxies. Our models, therefore, do not mimic real elliptical galaxies but only are expected to serve as basis for further detailed investigation. The parameters of the initial galaxy are $N = 5000$, radius $R = 45$ kpc, half-mass radius $R_h = 8.2$ kpc. We choose a system of units in which $G = M = 1$ where $G$ is the gravitational constant. If we take $M = 10^{11} M_\odot$ and the unit of length $L = 10$ kpc, then the unit of time turns out to be $4.7 \times 10^6$ yr and the unit of velocity is approximately 208 km/sec. 10 per cent of the mass of the galaxy is within 3 kpc and 90 per cent of the mass, within $2R_h$ implying the galaxy is a centrally concentrated one. The crossing time $t_{cr} = 2R_h / <\bar{v}^2>^{1/2} = 2.7$ units where $<\bar{v}^2>$ is the internal mean square velocity. The galaxy is close to virial equilibrium and possesses no net angular momentum.

Two similar galaxies are placed apart with an initial separation large enough so that at this distance the tidal force is negligible compared to the internal gravitational force in a system. An estimate of the tidal effects can be obtained from the ratio of the magnitude of the tidal force $F_T$ to the internal gravitational force $F_I$ assuming that the entire mass of each of the galaxy is concentrated at its respective centre. For equal mass galaxies, this gives rise to

$$\frac{F_T}{F_I} \approx 2\left(\frac{R}{D}\right)^3$$

(2)

where $R$ is the radius of the galaxy and $D$ the initial separation between them. We take the value of $F_T/F_I < 0.02$ to fix the value of $D$. One of the important parameter in a galaxy collision is the impact parameter (Saslaw 1985). The dynamically significant quantity corresponding to the impact parameter is the distance of closest approach $p$ of the orbit and its value is computed assuming the galaxies to be two point mass objects moving in Keplerian orbits. We have carried out numerical simulations of the encounter between the two galaxies for a range of values in $p$ such that $p/R_h = 0.5, 1.0, 2.0, 3.0, 3.5, 4.0, 5.0, 7.5$
Table 1. Collision parameters and results.

| Model | $p/R_h$ | $V_p/V_e$ | $\Delta U/|U|$ | $\Delta M/M$ | $(\Delta M/M)_e$ | $T_m/t_{fric}$ | $T_m/t_{cr}$ |
|-------|---------|-----------|----------------|--------------|-----------------|----------------|-------------|
| P1    | 0.5     | 0.482     | 1.031          | 0.173        | 0.055           | 1.9            | 4.6         |
| P2    | 1.0     | 0.575     | 1.040          | 0.153        | 0.043           | 3.1            | 8.1         |
| P3    | 2.0     | 1.023     | 1.045          | 0.162        | 0.044           | 7.7            | 14.8        |
| P4    | 3.0     | 1.102     | 1.034          | 0.162        | 0.047           | 16.9           | 59.9        |
| P5    | 3.5     | 1.214     | 0.137          |              | 0.036           |                |             |
| P6    | 4.0     | 1.435     | 0.104          | 0.026        |                 |                |             |
| P7    | 5.0     | 1.518     | 0.060          | 0.014        |                 |                |             |
| P8    | 7.5     | 1.731     | 0.031          | 0.004        |                 |                |             |
| P9    | 10.0    | 2.032     | 0.025          | 0.002        |                 |                |             |

Note: Column 1, model identification; column 2, values of the ratio $p/R_h$; column 3, the ratio $V_p/V_e$; column 4, relative change in energy; column 5, relative change in mass; column 6, the effective mass loss in merger remnants; column 7, ratio of the merging time $T_m/t_{fric}$; column 8, merging time in terms of crossing time.

and 10 and these models are respectively denoted by P1, P2, P3, ..... P9. This choice for the values of $p/R_h$ covers the border between merging and non-merging collisions. The interaction is expected to produce maximum tidal effects if the galaxies undergo collision on a marginally bound orbit and consequently we have taken the initial relative orbit to be parabolic. The X-Y plane coincides with the orbital plane. The computations are continued till the galaxies merged in a close collision or they remained detached and showed tendency to recede from each other in distant encounters. The integration of the equation of motion of each particle is carried out using Barnes’ treecode (Barnes & Hut, 1986) with a fixed time step $\Delta t = 0.02$. The accuracy of the treecode is controlled by the parameter $\theta$ which is the opening angle between two particles and it is chosen as $\theta = 0.7$. We use a softened potential for each particle in which the softening parameter ($\epsilon$) is about 100 pc which is less than the mean-free path of the particles in the system. With this choice of $\theta$ and $\epsilon$, the energy of the system is conserved to better than one per cent in all simulations. To see the dependence of our results on $\theta$ and $\epsilon$, we performed simulations making small changes in their values $\delta \theta$ and $\delta \epsilon$ and no significant differences were noticed in the end-results. We could not use large value for $N$ because of lack of computing facilities. The collision parameters and important results are given in Table 1.

3. Results and discussion

3.1 General features

In Table 1, models P1 - P4 represent closely interacting galaxies whereas models P5 - P9 represent distant collisions. In close collisions there is significant loss in orbital energy
Figure 1. Snapshot of particles in the orbital plane at selected times during the encounter. Horizontal panel represent models P1, P4, P5, and P8. The second figure in each row represents the pair of galaxies near the closest approach time.

As a result of which the orbit shrinks and the two galaxies ultimately merge. In distant collisions loss in orbital energy is negligible and the galaxies recede from each other after their closest approach time. The general features of the pair of galaxies are shown in Fig. 1.

This figure shows the projections of the particles in the orbital plane for selected times. In this figure, the snapshots of four representative models are given. The horizontal panels represent models P1, P4, P5 and P8 respectively from top to bottom. The second figure in each panel shows the galaxies very near to their close collision time. The first two are examples of merging models and the other two depict non-merging models. In model P1, merging occurs just after the first close approach time. Model P4 takes longer time for merging than model P1. In the non-merging models the galaxies more or less remain intact without causing much structural damage.
Figure 2. The separation of the galaxies is plotted as function of time. Left panel is for merging simulations and right panel for non-merging ones.

3.2 Features of merger remnant

The galaxies started off their collision on a parabolic orbit as a result of which the initial orbital energy is zero for all models. As the collision proceeds, orbital energy is being transferred to the internal energy of the galaxies and consequently the galaxies become bound by the closest approach time. Models P1 and P2 merge in less than $15t_{cr}$ and afterwards remain as a single system till the end of computation. Merging takes $40t_{cr}$ for model P3 and $60t_{cr}$ for model P4. Merging does not take place in last five models even after 75 crossing time. In these models the galaxies become completely detached by this time and their separations increase afterwards. The separations between the galaxies at various times are given in Fig. 2. The left panel shows the separations of the galaxies in the merging simulations and the right panel shows that of the non-merging models. In the merging simulations, the systems lose part of their orbital energy in the first encounter as a result of which the orbit shrinks and the internal energy increases. Subsequent passes through pericentre ultimately lead to merging. It may be noted that the merging time increases with $p/R_{h}$.

The amount of energy transferred from the orbit to the internal energy gives an idea about the strength of interaction. The ratio $\Delta U/|U|$ where $U$ is the unperturbed initial energy of a single galaxy and $\Delta U$, its change during an encounter indicate the possible outcome of an encounter. These values pertaining to a typical galaxy are given in Table 1. In this table $\Delta M/M$ represents the typical change in mass of a single galaxy during a collision. $\Delta U/|U| > 1$ for models P1 - P4 which implies that the galaxies undergo considerable disruption before merging. $\Delta M/M$ is less than 18 per cent for these merging models. Some of the particles leaving a particular galaxy are captured by the other galaxy and so the effective mass loss $(\Delta M/M)_{e}$ is about 5 per cent. The values of $(\Delta M/M)_{e}$ for merger remnants are given in Table 1. For all other models $\Delta U/|U| < 1$
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and \( \Delta M/M \) is less than 5 per cent implying that distant encounters do not produce considerable disruption in the structure of the colliding galaxies.

### 3.3 Merging criterion

The galaxies are moving with respect to their centre of mass in such a way that \( p < 2R \). This implies that the galaxies undergo overlapping collisions. A galaxy orbiting near another galaxy in an overlapping fashion experiences a drag force, viz. dynamical friction resulting in a steady deceleration in its motion. Orbital energy is transferred to the internal energy of the galaxies as a result of which the orbit decays and causes eventual merger of the two galaxies into a single system. Numerical and analytical estimates show for a bound system merging takes place in time scales that are short compared to the Hubble time. The merging time expressed in units of crossing time is given in eighth column of Table 1. One can also evaluate the merging time by using the formula

\[
 t_{\text{fric}} = \frac{1.17 r_i^2 v_c}{\ln \Lambda \ G M} \tag{3}
\]

(Binney & Tremaine 1987). Here \( r_i \) is the initial radius of the orbit and \( v_c \), the circular speed at a specific radius within which 50 per cent of the mass is concentrated. \( \ln \Lambda \) is known as the Coulomb logarithm and it is defined and tabulated in Binney & Tremaine (1987). The ratio \( T_m/t_{\text{fric}} \) for our merging models are given in seventh column of Table 1. It can be seen that the above formula underestimates the merging time and the difference increases with the distance of closest approach. The discrepancy is due to the fact that the above formula was derived under the assumption that the stellar systems move on circular orbits. Models P1 - P4 merge in less than \( 60t_{\text{cr}} \) and models P5 - P9 do not merge even after \( 75t_{\text{cr}} \). Earlier numerical simulations have shown that in a head-on collision, merging occurs when the collision velocity at minimum separation \( V_p < 1.16V_e \) where \( V_e \) is the escape velocity at that distance (Dekel et al. 1980). They used uniform density galaxy models containing less number of particles. The ratio \( V_p/V_e \) for all the models are shown in table 1. Our simulation show that merging occurs for values of \( V_p/V_e < 1.2 \) and it agrees with the results obtained by previous workers. White (1978) has shown that merging of spherical galaxies takes place when the galaxies overlap significantly at closest approach such that \( p < 2.5R_h \). An inspection of the values of \( p/R_h \) in table 1 shows that for centrally concentrated galaxies merging takes place when \( p < 3R_h \). This implies that a knowledge of the values of \( p/R_h \) together with \( V_p/V_e \) in principle will determine the fate of a galactic collision.

Several workers have shown that close collisions of two galaxies result in the merger if the collision velocity is less than the internal velocity dispersion of the galaxies. Binney & Tremaine (1987) have shown that the fate of collisions of two non-rotating spherical galaxies can be determined by two quantities \( \dot{E} \) and \( \dot{L} \) of the initial model where \( \dot{E} = E_{\text{orb}}/(0.5 < v^2 >) \) and \( \dot{L} = L_{\text{orb}}/(R_h < v^2 >^{1/2}) \) and \( E_{\text{orb}} \) and \( L_{\text{orb}} \) are the energy and angular momentum per unit mass respectively and \( < v^2 > \) is the internal mean square
velocity. The evolution of galaxies in the \((\hat{E}, \hat{L})\) plane at four times during the collision are shown in Fig. 3. In this figure the solid curve represents the locus of circular orbits with energy \(\hat{E}\) and angular momentum \(\hat{L}\). The dashed curve divides the \((\hat{E}, \hat{L})\) space into two regions viz. merging and non-merging. In the lower region below the dashed curve, merging occurs rapidly for bound orbits and within a Hubble time for unbound ones. In the upper portion merging occurs more slowly or not at all. The ordinate passing through the origin is the locus of parabolic orbit encounters. Initially at \(T = 0\), all the models lie along this line, two of them within the merging region and the rest of them in the non-merging part. At \(T = 40\), models P1, P2 and P3 are within the merging region and P1 has already moved into the rapid merging part showing that merging is complete in that model. At \(T = 80\) models P2 and P3 also complete merging into a single system and model P4 is just on the boundary of these two regions. At the end of the computation model P4 also completes merging. Models P5 - P9 do not enter the merging region. According to Binney & Tremaine (1987) rapid merging occurs for pairs of galaxies with \(\hat{L} \leq 3.5\). However our experiments show that for more centrally concentrated galaxies,
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Figure 4. The surface density of the merger remnants. The solid line represents the best fit for $r^{1/4}$ law and the filled circles are points obtained from numerical simulations.

merging occurs for values of $\hat{L} \leq 4.5$. This suggests that for centrally concentrated galaxies the dashed line in the above figure should be pushed upward to accommodate all galaxies that merge in less than a Hubble time. The discrepancy may be due to the different types of models used for the galaxies.

3.4 Density distribution of merger remnants

Parabolic encounters in which the galaxies have comparable mass are expected to produce maximum tidal distension in the outer parts of the density profile of the merger remnant. White (1978) has shown that more particles escape from parabolically colliding galaxies than from those which are initially in bound orbits. McGlynn (1990) has noted that the presence of a steady tidal field in bound orbits produces less catastrophic effects than when the system is in non-bound orbits. This is due to the fact that the tidal forces are equal in magnitude and opposite in direction at diametrically opposite points in the orbit as a result of which the effects at these points cancel each other. Highly eccentric orbit encounters are not expected to produce significant rearrangement within the components.
Burkert (1993) studied the surface brightness distribution of a large sample of elliptical galaxies and showed that de Vaucouleurs law provides an excellent fit to the observed brightness distribution for all galaxies within the radius range $0.1 R_e \leq R \leq 1.5 R_e$ where $R_e$ represents the effective radius of the galaxy that contains half the total luminosity. We have tried to fit the surface density profiles of the merger remnants to a de Vaucouleurs $r^{1/4}$ law

$$\log I(r) = \log I(0) - 3.33[(r/r_e)^{1/4} - 1]$$

(4)

where $r_e$ is the radius containing half the total light and $I(0)$ is the brightness at $r_e$. The fitted curve and the points obtained from the simulations are shown in Fig. 4. In this figure the filled circles represent the points obtained from simulations and the best fit is represented by the solid line. The fit is remarkably good in the inner regions where more than 80 per cent of mass of the remnant lies. Departures from $r^{1/4}$ law occur beyond $6 R_h$. Our results are in agreement with those of earlier workers (Kormendy, 1977; Capelato et al., 1995 etc) who have noted the formation of a bump in the outer parts which disappeared gradually as the remnant approached an $r^{1/4}$ profile. In our models departure from an $r^{1/4}$ law becomes increasingly evident as the ratio $p/R_h$ approaches the value three. The disappearance of the bump could not be observed as we did not follow the evolution of the remnants beyond merging. Our results point out the fact that some tidally distended galaxies may possibly have formed by the merger between galaxies of comparable mass and size.

White (1978) considered collisions of a spherical galaxy with either a single point mass or a second galaxy of similar type. He observed the development of a de Vaucouleurs profile in the merger remnants with tidal distension in the outer parts. The merging simulations of Navarro (1989, 1990) involving equal mass galaxies demonstrated the robustness of $r^{1/4}$ law in describing the density profiles of merger remnants. He also noted the development of an excess light in the outer parts characteristic of tidally distended galaxies. McGlynn (1990) studied the strong tidal interactions between galaxies having different masses and pericentric distance in the range 0 - 20. The simulations in which the victims survived showed density profiles $\rho \propto r^{-4}$. Gonzalez-Gracia & van Albada (2005) performed a large number of numerical simulations involving spherical galaxies with various mass ratios, impact parameters and orbital energies. The surface density profiles of the merger remnants followed the de Vaucouleurs law for about 10 mag. Deviation from $r^{1/4}$ law started appearing at large distance from the centre and became prominent when the mass ratio approached unity. Our results agree with those obtained by Navarro (1989, 1990) and Gonzalez-Gracia & van Albada (2005). A detailed comparison with other two works becomes difficult because of the difference in methods and the type of galaxies used in the simulation.

4. Conclusions

Numerical simulations of both merging and non-merging collisions of equal mass non-rotating galaxies have been performed to investigate the effect of changing the impact
parameter. The model galaxies presented in the simulations consist of the stellar component only. They, therefore, are not expected to be perfect models of realistic elliptical galaxies. Our results are valid in the case of centrally concentrated galaxies whose density distribution closely follows that of a polytrope of index $n = 4$ and the relative orbit of collision is parabolic. The important results of the present simulations are the following.

(1) Merging takes place in less than a Hubble time for closely colliding galaxies in which $p \leq 3R_h$ and $V_p \leq 1.2V_e$.

(2) Merging does not take place during the first close contact but during the subsequent close contact only. The merging time increases with the distance of closest approach $p/R_h$.

(3) The fate of a collision can fairly well be predicted by computing the relevant values of $\hat{E}$ and $\hat{L}$ and plotting them in the $(\hat{E}, \hat{L})$ plane. Merging takes place for values of $\hat{L} \leq 4.5$.

(4) The density distribution of the merger remnant can be fitted by de Vaucouleurs $r^{1/4}$ law in the inner parts up to $R = 6R_h$. The fit is remarkably good in this region and the density profiles show tidal distension in the outer parts. The tendency to form tidal distension increases with the distance of closest approach.

Results (1) and (2) have been obtained by previous workers for uniform spherical galaxies. The present work extends these results to the case of centrally concentrated galaxies. Results (3) and (4) of the present work appear to be new.

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References

Fall, S. M., 1981, in The Structure & Evolution of Normal Galaxies, eds. S. M. Fall & D. Lynden-Bell, University of Cambridge, p. 1
Naab, T., & Burkert, A., 2000, in Dynamics of Galaxies From the Early Universe to the Present, ed. F. Combes, G. A. Mamon, & V. Charmandaris (San Francisco) ASP Conf. Ser., 197, 267