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Optimal long binary phase code-mismatched filter pairs with applications to ionospheric radars

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Abstract. Radars have been used extensively for ionospheric studies. In an ionospheric radar measurement one usually implements different types of codes and the choice largely depends on the geophysical phenomenon under investigation. As a result many kinds of coding and decoding methods have been used including binary phase codes. In ionospheric physics the performance of binary phase codes are usually investigated in terms of spatial and temporal resolution. This is done usually by employing a matched filter, which creates unwanted side lobes at the output of the receiver. These side lobes can be eliminated by using a mismatched filter. But there is an associated loss in signal-to-noise ratio (**SNR**). In this paper we have presented long optimal binary phase code-mismatched filter pairs that may be used in several applications including ionospheric radar measurements. This was done by investigating 1.04×10^9 number of binary phase codes.

Keywords : plasmas – data analysis – radar astronomy

1. Introduction

Binary phase codes have been often used in radar systems. The most widely known binary phase codes are Barker codes (Barker 1953). Other families of binary phase codes, which are called alternating codes, have been also discovered (Lehtinen & Häggström 1987). These and other kinds of codes are used to vary the carrier signals from a radar transmitter in accordance with their waveforms. In the radar receiver information about the target is obtained by employing a suitable filter. In binary phase-coded radar measurements a

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matched filter is usually used to obtain a very high range resolution without decreasing the average transmitted power. However, matched filtering of a binary phase code gives unwanted sidelobes at the filter output. The amplitude of the sidelobes depends on the phase patterns of the binary phase code. Significant research effort has been done to search binary phase patterns that give smallest possible sidelobes. Most often peak-to-sidelobe ratio (PSR), integrated sidelobe ratio (ISR) and merit factor (F) are used as criteria to measure the performance of binary phase radar waveforms. For example, Barker codes have relatively high PSR. Other kind of binary phases with improved PSR have been found, including the 28-element code by Turyn (1976) and the 40-element code by Lindner (1975).

Although binary phase codes with maximum PSR can be satisfactory for some applications, in some cases removing the sidelobes reveals new and important information. Key et al. (1959) showed that weighting networks to be placed after the standard matched filter can be designed which reduces the sidelobes to an arbitrary low level. For any periodic digital signal with linearly independent cyclical shifts, Ipatov (1977) has shown that a filter can be constructed that suppresses to a zero level all the sidelobes. However, the filter has associated **SNR** losses when compared to the matched filter. Ipatov (1980) carried out a computer search for a binary periodic signal-filter pair with minimum possible **SNR** losses. The search includes all binary codes of length up to 30 elements. A different approach of eliminating the sidelobes in periodical binary phase codes by using mismatched filter have been published by Rohling and Plagg (1989). Exhaustive search for optimal aperiodic binary phase codes and mismatched filter pairs up to length of 25 has been carried out by Lehtinen et al. (2004). The benefits of eliminating sidelobes have also been demonstrated in practice by analyzing ionospheric radar measurements.

In this paper we extend the investigation of binary phase codes in Lehtinen et al. (2004) up to a length of 30. A computer search is carried out to find aperiodic binary phase code-mismatched filter pairs with minimum possible **SNR** losses. We first formulate mismatched filtering operations and then describe our criterion for selecting an optimal binary phase code-mismatched filter pair. Finally, we present the search results.

2. Mismatched filtering of aperiodic binary phase codes

Since our investigations are concerned with a digital system, it is convenient to formulate the problem in a discrete form. This means that we follow the formulation in Damtie et al. (2007). We shall represent the phase pattern of a binary phase code by $\epsilon(n)$ and waveforms of the corresponding matched and mismatched filters are denoted by $h_m(n)$ and $\lambda(n)$, respectively. Then the signal at the output of the matched filter $S^o_{h_m}(n)$ may be given by

$$S^o_{h_m}(n) = \epsilon(n) * h_m(n), \tag{1}$$

and similarly the signal at the output of the mismatched filter $S^o_\lambda(n)$ can be expressed by

$$S^o_\lambda(n) = \epsilon(n) * \lambda(n). \tag{2}$$

Here $*$ denotes convolution. The impulse response of a matched filter is the inverse replica of the input signal and hence $h_m(n) = \epsilon(-n)$. The impulse response of a mismatched filter $\lambda(n)$, which does not create unwanted sidelobes, can be calculated from Damtie et al. (2007)

$$\lambda(n) = \frac{1}{2\pi} \int_0^{2\pi} \frac{N}{\varepsilon(\omega)} e^{-jn\omega} d\omega, \tag{3}$$

where N is the length of the binary code, $\varepsilon(\omega)$ and $N/\varepsilon(\omega)$ are the discrete Fourier transforms of $\epsilon(n)$ and $\lambda(n)$, respectively. When $\epsilon(n)$ is convolved with the mismatched filter calculated from eq. (3), one obtains a single peak without any sidelobes. Mismatched filtering requires that the Fourier transform of the impulse response of the coding filter should not have zeros.

Table 1. Optimal binary phase codes.

Optimal binary phase pattern	N	R_N
+++--+++------+-+-+--+--+-	26	0.877
+++----+++-+++-+++-++-+--+-	27	0.862
++---+++-------+-+-++-++--+-	28	0.847
+-++--+--+-+-+-------+++--+++	29	0.853
+++++-----+--+--+-+-+---++---+	30	0.864
++---++---+-+-+--+--+------++++	31	0.860
+-+-+-+--+-++----++++++++--++--+	32	0.843
+-+-+-++--++-++-++++-++++------++	33	0.856
++++++++----++-+--+-+-+-+--++--++-	34	0.867
+-+-+--+-+++++++-+--++-+---+++--++-	35	0.851
+--+-++-++--------++---+-++---+-+-+-	36	0.847
++++++++----++++--+-++-+-+-+--++--++-	37	0.850
++--++--+++++++++----++-+--+-++-+-+-+-	38	0.855
++++++++--+--+-++-++---++++---++-+-+-+-	39	0.849
++++++++-++-+--+-+--++--++-----++++---+-+-+-+	45	0.850
++++++++--++--++--++++----+++-++-+--+-++-+-+-+-+-+	50	0.843
-+-------++--+---+---++++++--+-++--+--+-+-++++--+++--+-+--++	60	0.850
-------+++++++--+++-++++--+++-++-++--+++-+--++++--+-+++-+ --+-++-+-+-+-	70	0.845
--++-++-----++-++-----+----+--++----+--+-+--+-+-+---+-+-++-++++ ++--++-----++---+	80	0.820
-+-++-++-++-+--++--+-+----+++--++-+-++++-++--+++-+++-----++- ----+++-+-++++++--+---+-+++++++++-+-+-++	100	0.823

3. Optimal binary phase codes and mismatched filter pairs

The selection of optimal binary phase code and mismatched filter pair is based on **SNR** performance. A matched filter gives the maximum possible output **SNR** when one has an input signal corrupted by white noise. Therefore, we shall measure the **SNR** performance of a binary phase code and mismatched filter pair by comparing it to the corresponding output **SNR** from a matched filter. We introduce a parameter R_N

$$R_N = \frac{\mathbf{SNR_{mis}}}{\mathbf{SNR_{mat}}}, \tag{4}$$

where $\mathbf{SNR_{mis}}$ is the **SNR** at the output of a mismatched filter and $\mathbf{SNR_{mat}}$ is the **SNR** at the output of a matched filter. By considering white Guassian noise at the input and combining eqs (1–4), one can obtain

$$R_N = \sum_{n=-\infty}^{\infty} |h_m(n)|^2 \left(\sum_{n=-\infty}^{\infty} |\lambda(n)|^2 \right)^{-1}. \tag{5}$$

For a given length N of a binary code, we have investigated the **SNR** performance of different phase patterns and choose a pattern with an optimal value of R_N.

In this work for $26 \leq N \leq 30$, the values of R_N have been calculated for all possible 2^{N-1} number of binary phase code-mismatched filter pairs and the optimal binary phase code is the one that gives the maximum possible value of R_N. Table 1 shows the phase patterns and the values of R_N of the optimal binary phase codes we have found. When all the optimal binary phase codes are compared with each other, one can see that the 26-bit length binary phase code with phase $+++--+++------+-+-+--+--+-$ has the maximum value of R_N.

It is extremely time consuming computing for some codes and for very long ones impractically to carry out exhaustive computer search for binary phase codes with $N \geq 31$. However, we can get a glimpse of R_N values for long codes by carrying out random search. The results obtained by investigating 1000 randomly selected codes for each given length of a code are shown in Table 1 and we see that performance did not increase with length.

4. Conclusions

We have shown the optimal binary phase codes found by investigating 1.04×10^9 number of binary phase code-mismatched filter pairs. These codes may be used in several applications. Some ionospheric physics can be better understood by having a very high spatial and temporal resolutions. The characteristics of binary codes presented in Table 1 may be exploited to carry out a very high resolution ionospheric radar measurements.

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