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Stationary solutions and their stability in the magnetic-binary problem when the bigger primary is a triaxial rigid body

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Abstract. This paper deals with the stationary-state in the planer magnetic-binary problem when the bigger primary is a triaxial rigid body with its equatorial plane coincident with the plane of motion by taking different values of semiaxes of the triaxial rigid body. There exists five or seven equilibrium points (L_1, L_2, L_3 are collinear and L_4, L_5, L_6, L_7 are non-collinear) corresponding to certain values of mass parameter μ and magnetic parameter λ, the libration point L_1 does not exist when $\lambda = 0, 2, 3$ the libration point L_2 does not exist when $\lambda = 2, 3$. We have also found that only L_2 is stable for given values of μ and λ.

Keywords : advanced dynamics, magnetic-binary-problem, equilibrium points (stationary points or libration points), stability.

1. Introduction

Many contributions have been made to the problem of the motion of a charged particle in a magnetic-dipole field by Stormer (1907), De Vojelaere (1949, 1950, 1958), Graef and Kusaka (1938) and Dragt (1965). Goudas and Petsagourakis (1985) have studied the motion of a charged particle P of charge q and mass m which is moving under the magnetic forces due to two dipoles of magnetic moments $\bar{M}_1$ and $\bar{M}_2$. These dipoles participate in circular motion of their carrier stars S_1 and S_2 which move around their centre of mass.

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Mavraganis (1979) investigated the influence of oblate primaries on the stationary - state of the plane's magnetic binary problem. Bhatnagar et al. (1993) have studied the same problem including the effect of gravitational forces taking both the primaries as spheres. In this paper we generalize this problem considering the bigger primary as a triaxial rigid body.

We have considered a case of Earth-Moon system. In this system the Earth has been taken as a triaxial rigid body, as earth's equator is an ellipse and not a circle. The semi axes a_1, a_2, a_3 of the earth are assumed to be in the neighbourhood of 6400 km.

We may note that though there seems to be very little change in the location of equilibrium points in dimensionless distance but in physical dimension these changes are of the order of kilometers. Though we have applied the theory so developed to only one system viz Earth-Moon system, it can be applied to any other systems also.

Due to the motion of the primaries under the influence of their mutual attractions, the magnetic field of each one varies in space and time, hence the particle is moving in a sufficiently complicated field. Since the gravitational force acting on the particle is evidently very small, it is neglected. In order to simplify the problem under its new consideration, we suppose: the primaries describe circular orbits with constant angular velocity (the mean motion), the sum of their masses is equal to unity and the distance between them is constant (unity). The equations of motion and an integral (energy) of -motion are derived in the rotating co-ordinate system, using dimensionless variables. Stability of stationary points (also called libration points) is discussed by examining the roots of the characteristic equation.

2. Equation of motion

Let us consider two masses m_1 and m_2 at P_1 and P_2 carrying dipoles moving in circular orbits without rotation around their centre of mass 'O' with angular velocity n. Let there be a particle P of charge q and mass m_3 moving in the plane of motion of m_1 and m_2 and is being influenced by their motion but not influencing them. We consider the bigger primary of mass m_1 to be a triaxial rigid body with one of its axes as the axis of symmetry whose equatorial plane coincides with the plane of motion. Let the position vector of P at any time t be $\bar{r}$ with respect to a rotating frame of reference $O(x, y, z)$ which is rotating with the same angular velocity as those of masses m_1 and m_2. Suppose initially the masses m_1 and m_2 lie on the x-axis. Since the angular velocity of the frame of reference $O(x, y, z)$ is the same as those of the masses m_1 and m_2 and at $t = 0$ they lie on the x-axis, they will continue to stay at rest on the x-axis. We choose the distance between the masses m_1 and m_2 as the unit of distance and the sum of their masses as the unit of mass. We choose the unit of time in such a way that the gravitational constant G has the value unity. We further choose the unit of charge such that $q/m_3c = 1$. Since the gravitational force acting on the particle is very small as compared to Lorentz forces, we may neglect it.

The equation of motion of the charged particle P of mass m_3 in the synodic system is

$$m_3\left[\frac{\partial^2\bar{r}}{\partial t^2}+2\bar{\omega}\times\frac{\partial\bar{r}}{\partial t}+\bar{\omega}\times(\bar{\omega}\times\bar{r})\right]=\bar{F}_1+\bar{F}_2, \tag{1}$$

where $\bar{\omega}=\omega_1 \overset{\sqcup}{i} +\omega_2 \overset{\sqcup}{j} +\omega_3 \overset{\sqcup}{k}=$ angular velocity vector, Since the axis of rotation is about z-axis therefore components of angular velocity about x- and y-axis are zero, i.e.

$$\begin{aligned}
\bar{\omega} &= \omega_3 \overset{\sqcup}{k}= n \overset{\sqcup}{k} \text{ say} = \text{constant},\\
\bar{F}_1 &= -\frac{q}{c}\frac{d\bar{A}_1}{dt}+\frac{q}{c}\left[\frac{d\bar{r}}{dt}\times(\bar{\nabla}\times\bar{A}_1)\right],\\
\bar{F}_2 &= -\frac{q}{c}\frac{d\bar{A}_2}{dt}+\frac{q}{c}\left[\frac{d\bar{r}}{dt}\times(\bar{\nabla}\times\bar{A}_2)\right],\\
c &= \text{velocity of light},\\
\bar{A}_1 &= \frac{\mu_0\bar{J}_{01}}{4\pi r_1}\left[V_1+\frac{1}{2r_1^2\rho_1}(I_1+I_2+I_3-3I)\right],\\
\bar{A}_2 &= \frac{\mu_0\bar{J}_{02}V_2}{4\pi r_2}
\end{aligned}$$

$\bar{J}_{01}$ and $\bar{J}_{02}$ are the average values of the current density vectors $\bar{J}_1(\bar{r})$ and $\bar{J}_2(\bar{r}$ respectively,

V_1 and V_2 are the volumes of the bigger and smaller primaries respectively, ρ_1 and m_1 are the density and mass of the bigger primary, I_1, I_2, I_3 are the principal moments of inertia of the bigger primary with its mass m_1 about its centre of mass,

$$\begin{aligned}
I_1 &= \frac{b_1^2+c_1^2}{5R^2}m_1\\
I_2 &= \frac{a_1^2+c_1^2}{5R^2}m_1\\
I_3 &= \frac{a_1^2+b_1^2}{5R^2}m_1
\end{aligned}$$

I is the moment of inertia about the line joining the centre of mass of the bigger primary of mass m_1 and the infinitesimal body of mass m_3 i.e. about $\overrightarrow{PP_1}$

$$I=I_1l_1^2+I_2m_1^2+I_3n_1^2$$

I_1, m_1, n_1 are the direction cosines of $\overrightarrow{PP_1}$, R is the dimensional distance between m_1 and m_2. a_1, b_1, c_1 are the lengths of the semi axes of the bigger primary.

Here we have taken

$$\frac{\mu_0\overline{J}_{01}V_1}{4\pi}=\overline{M}_1$$

and

$$\frac{\mu_0 \overline{J}_{02} V_2}{4\pi} = \overline{M}_2.$$

Let us suppose that $\overline{M}_1$ has the components (0,1) and $\overline{M}_2$ has the components $(0, \lambda)$.

Now the equations of Motion (1) in the Cartesian Synodic System become

$$\begin{aligned} \ddot{x} - S\dot{y} &= U_x, \\ \ddot{y} + S\dot{x} &= U_y, \end{aligned}$$

where

$$\begin{aligned} U &= \frac{n^2(x^2+y^2)}{2} + nx\left\{\frac{1}{r_1} + \frac{\lambda}{r_2} + \frac{1}{2r_1^3}(2\sigma_1 - \sigma_2) - \frac{3y^2}{2r_1^5}(\sigma_1 - \sigma_2)\right\}, \\ S &= 2n - (x-\mu)\left\{\frac{1}{r_1^3} + \frac{3(2\sigma_1 - \sigma_2)}{2r_1^5} - \frac{15y^2(\sigma_1 - \sigma_2)}{2r_1^7}\right\} - \frac{\lambda(x+1-\mu)}{r_2^3}, \\ r_1^2 &= (x-\mu)^2 + y^2, \\ r_2^2 &= (x+1-\mu)^2 + y^2, \\ \mu &= \frac{m_2}{m_1+m_2} \leq \frac{1}{2}, \\ \sigma_1 &= \frac{a_1^2 - c_1^2}{5R^2}, \quad \sigma_2 = \frac{b_1^2 - c_1^2}{5R^2}, \quad \sigma_1, \sigma_2 \ll 1, \\ n^2 &= 1 + \frac{3}{2}(2\sigma_1 - \sigma_2). \end{aligned}$$

3. Investigation of the equilibrium points

We know that the equilibrium points are the solutions of the equations

$$U_x = 0 = U_y, \tag{2}$$

We group the solutions of Equations (2) into two kinds; those with $y = 0$ (the collinear equilibrium points) and those with $y \neq 0$ (non-collinear equilibrium points) these two kinds of points are examined separately.

For $y = 0$, the roots of the equation $U_x = 0$, therefore determine the abscissa of the collinear points. We investigate the existence and location of the collinear points into the following three intervals $I_1(\mu - 2, \mu - 1), I_2(\mu - 1, 0)$ and $I_3(\mu, \mu + 1)$ on the x-axis. For $y \neq 0$ (non-collinear equilibrium points) we solved the equations $U_x = 0 = U_y$ numerically in all the cases for $\mu = 0.0121$ and $\lambda = -3$. We have found that there exists four non-collinear points in some cases.

3.1 The stability of the equilibrium points

Let (x_0, y_0) be the coordinates of any one of the equilibrium point. Let ξ_0, η_0 denote small displacement of the particle from equilibrium point. Then, near these points the particle moves according to the following set of non-linear (variation) equations.

$$\begin{aligned}
\ddot{\xi}_0 - \dot{\eta}_0(S + S_x\xi + S_y\eta)_0 &= \xi_0\left(\frac{\partial^2 U}{\partial x^2}\right)_0 + \eta_0\left(\frac{\partial^2 U}{\partial x \partial y}\right)_0, \\
\ddot{\eta}_0 + \dot{\xi}_0(S + S_x\xi + S_y\eta)_0 &= \xi_0\left(\frac{\partial^2 U}{\partial x \partial y}\right)_0 + \eta_0\left(\frac{\partial^2 U}{\partial y^2}\right)_0.
\end{aligned} \tag{3}$$

Linearizing Equation (3), since the non-linear terms divided by the norm of the vector $T(\xi, \eta, \dot{\xi}, \dot{\eta})$ are negligible when this norm tends to zero, i.e.

$$\lim_{\|T\|\to 0} \frac{h\dot{\xi}}{\|T\|} = 0,$$

$$\lim_{\|T\|\to 0} \frac{h\dot{\eta}}{\|T\|} = 0,$$

$$h = S_x\xi + S_y\eta$$

We get the following system of second order equations with constant coefficients

$$\begin{aligned}
\ddot{\xi}_0 - \dot{\eta}_0 S_0 &= \xi_0\left(\frac{\partial^2 U}{\partial x^2}\right)_0 + \eta_0\left(\frac{\partial^2 U}{\partial x \partial y}\right)_0, \\
\ddot{\eta}_0 + \dot{\xi}_0 S_0 &= \xi_0\left(\frac{\partial^2 U}{\partial x \partial y}\right)_0 + \eta_0\left(\frac{\partial^2 U}{\partial y^2}\right)_0,
\end{aligned} \tag{4}$$

with

$$\begin{aligned}
\left(\frac{\partial^2 U}{\partial x^2}\right)_0 = {} & n^2 - \frac{n(x_0-\mu)}{r_1^3}\left\{2 + \frac{3(2\sigma_1-\sigma_2)}{r_1^2} - \frac{15(y_0)^2(\sigma_1-\sigma_2)}{r_1^4}\right\} \\
& - \frac{2\lambda n(x_0+1-\mu)}{r_2^3} \\
& + n x_0\left[\frac{3(x_0-\mu)^2}{r_1^5}\left\{1 + \frac{5(2\sigma_1-\sigma_2)}{2r_1^2} - \frac{35(y_0)^2(\sigma_1-\sigma_2)}{2r_1^4}\right\}\right. \\
& \left. - \frac{1}{r_1^3}\left\{1 + \frac{3(2\sigma_1-\sigma_2)}{2r_1^2} - \frac{15(y_0)^2(\sigma_1-\sigma_2)}{2r_1^4}\right\}\right] \\
& + n x_0 \lambda\left\{\frac{3(x_0+1-\mu)^2}{r_2^5} - \frac{1}{r_2^3}\right\}
\end{aligned}$$

$$\left(\frac{\partial^2 U}{\partial x \partial y}\right)_0 = -\frac{n y_0}{r_1^3}\left\{1 + \frac{3(2\sigma_1-\sigma_2)}{2r_1^2} + \frac{3(\sigma_1-\sigma_2)}{r_1^2} - \frac{15(y_0)^2(\sigma_1-\sigma_2)}{2r_1^4}\right\} - \frac{n\lambda y_0}{r_2^3}$$

$$+\frac{3nx_0(x_0-\mu)y_0}{r_1^5}\left\{1+\frac{5(2\sigma_1-\sigma_2)}{2r_1^2}+\frac{5(\sigma_1-\sigma_2)}{r_1^2}-\frac{35(y_0)^2(\sigma_1-\sigma_2)}{2r_1^4}\right\}$$
$$+\frac{3n\lambda x_0y_0(x_0+1-\mu)}{r_2^5}$$

and

$$\left(\frac{\partial^2U}{\partial y^2}\right)_0 = n^2+nx_0\left[\frac{3(y_0)^2}{r_1^5}\left\{1+\frac{5(2\sigma_1-\sigma_2)}{2r_1^2}+\frac{25(\sigma_1-\sigma_2)}{2r_1^2}-\frac{35(y_0)^2(\sigma_1-\sigma_2)}{2r_1^4}\right\}\right.$$
$$\left.-\frac{1}{r_1^3}\left\{1+\frac{3(2\sigma_1-\sigma_2)}{2r_1^2}-\frac{3(\sigma_1-\sigma_2)}{r_1^2}\right\}\right]+nx_0\lambda\left\{\frac{3(y_0)^2}{r_2^5}-\frac{1}{r_2^3}\right\}$$

by means of which we can study the stability of motion near the equilibrium points, here r_1, r_2 to be calculated at $x = x_0$, $y = y_0$.

The characteristic equation of Equation (4) is

$$\Lambda^4 + B\Lambda^2 + D = 0 \tag{5}$$

where

$$B = S^2 - \left(\frac{\partial^2U}{\partial x^2}\right)_0 - \left(\frac{\partial^2U}{\partial y^2}\right)_0,$$
$$D = \left(\frac{\partial^2U}{\partial x^2}\right)_0\left(\frac{\partial^2U}{\partial y^2}\right)_0 - \left\{\left(\frac{\partial^2U}{\partial x\partial y}\right)_0\right\}^2.$$

The equilibrium points are stable when all the roots of the characteristic Equation (5) are pure imaginary and this is possible only if

$$\Lambda^2 = \frac{-B \pm \sqrt{B^2-4D}}{2}$$

gives −ve values,

it implies

(i) $B^2 - 4D > 0, B > 0$ and
$B > \sqrt{B^2-4D} \Rightarrow D > 0$

(ii) $B^2 - 4D = 0$ and $B > 0$.

4. Numerical results

We calculated the collinear and non-collinear equilibrium points by giving small perturbations in the semi axes of the triaxial rigid body (earth) in the cases when $\lambda = -2, 0, 2, 3$

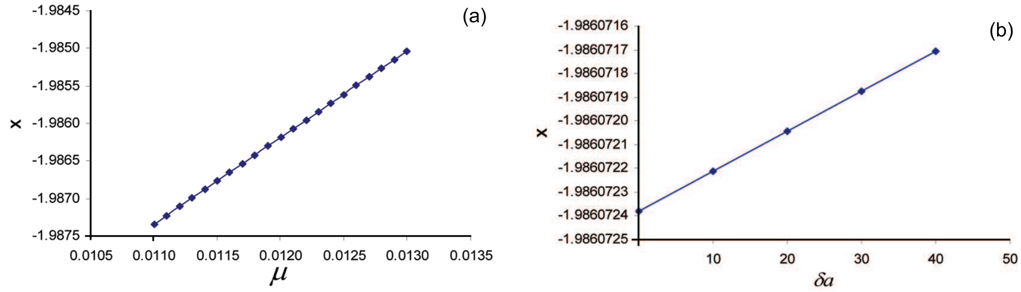

Figure 1. Libration point L_1 for $(\lambda = -2)$.

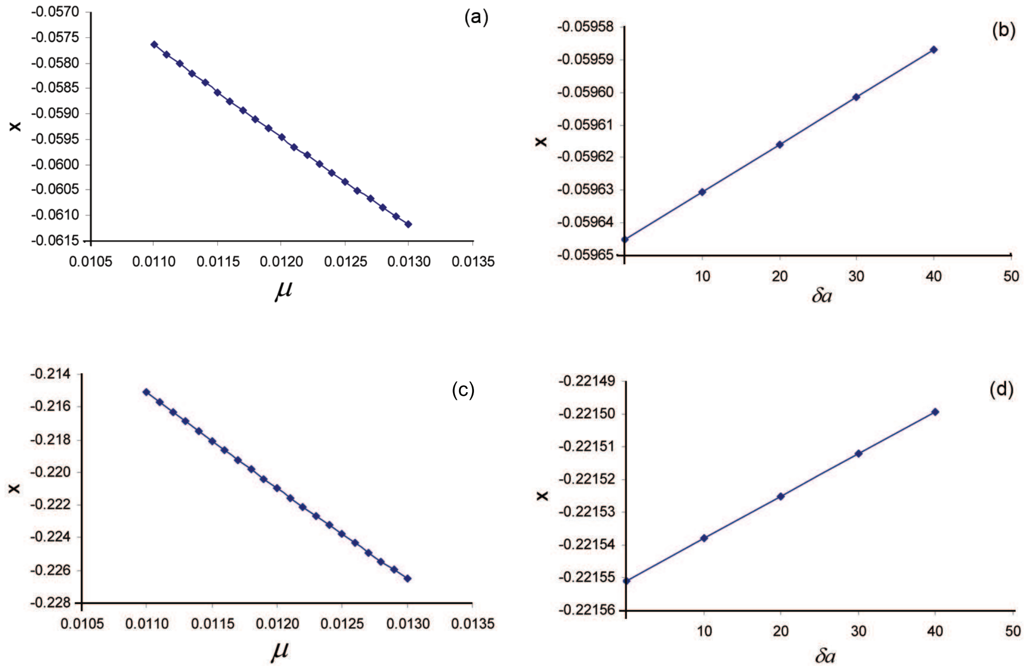

Figure 2. (a), (b) Libration point L_2 for $(\lambda = -2)$; (c), (d) Libration point L_2 for $(\lambda = 0)$.

(See Table 1-8). The collinear libration points are also shown graphically in figures (1a-b, 2a-d, 3a-f). Figure 1a shows the location of equilibrium point $L_1(\lambda = -2)$ for different values of μ. Figure 1b shows the location of equilibrium point $L_1(\lambda = -2)$ for different values of δa (change in the semi axis 'a' of the trixial body for $\mu = 0.0121$). Figures 2a and 2c show the location of equilibrium point $L_2(\lambda = -2, 0)$ for different values of μ. Figures 2b and 2d show the location of equilibrium point $L_2(\lambda = -2, 0)$ for different values of δa (change in the semi axis 'a' of the trixial body for $\mu = 0.0121$). Figures 3a, 3c and 3e show the location of equilibrium point $L_3(\lambda = -2, 0, 2)$ for different values of μ. Figures 3b, 3d and 3f show the location of equilibrium point $L_3(\lambda = -2, 0, 2)$ for different values of δa (change in the semi axis 'a' of the trixial body for $\mu = 0.0121$). Table 9 gives the characteristic roots of L_i, $i = 1, \ldots, 5$ for ($\mu = 0.0121$). This table indicates that only L_2 satisfies the conditions of stability for given value of μ and λ. All other libration points viz. are unstable.

Table 1. Semi-axes of the triaxial rigid body.

Parameter	Case 1	Case 2	Case 3	Case 4	Case 5
α_1	6400	6400	6400	6400	6400
α_2	6400	6390	6380	6370	6360
α_3	6400	6380	6360	6340	6320
σ_1	0	3.46×10^{-7}	6.908×10^{-7}	1.0345×10^{-6}	1.377×10^{-6}
σ_2	0	1.728×10^{-7}	3.449×10^{-7}	5.161×10^{-7}	6.865×10^{-7}

where a_1, a_2, a_3 are the lengths of semi-axes of triaxial body. Here we took five different cases of different semi-axes of triaxial rigid body. With respect to these five cases we calculated all the libration points and studied their stability.

We may note that in all the above cases, the libration point L_1 lies within the interval $I_1(\mu - 2, \mu - 1)$ when $\lambda = -2$ but does not exist when $\lambda = 0, 2, 3$.

We may note from Tables 3 and 4 that in all the above cases, the libration point L_2 lies within the interval $I_2(\mu - 1, 0)$ when $\lambda = -2$ and 0 but does not exist when $\lambda = 2, 3$.

We may note from Tables 5, 6 and 7 that in all the above cases, the libration point L_3 lies within the interval $I_3(\mu, \mu + 1)$ when $\lambda = -2, 0$ and 2 but does not exist when $\lambda = 3$.

We may note that in each of the above cases there exist four non-collinear libration points.

Here we note that in all the above cases, only the libration point L_2 gives the pure imaginary characteristic roots. Hence it is stable. All other libration points viz. $L_i, i = 1, 3, 4, 5, 6, 7$ are unstable.

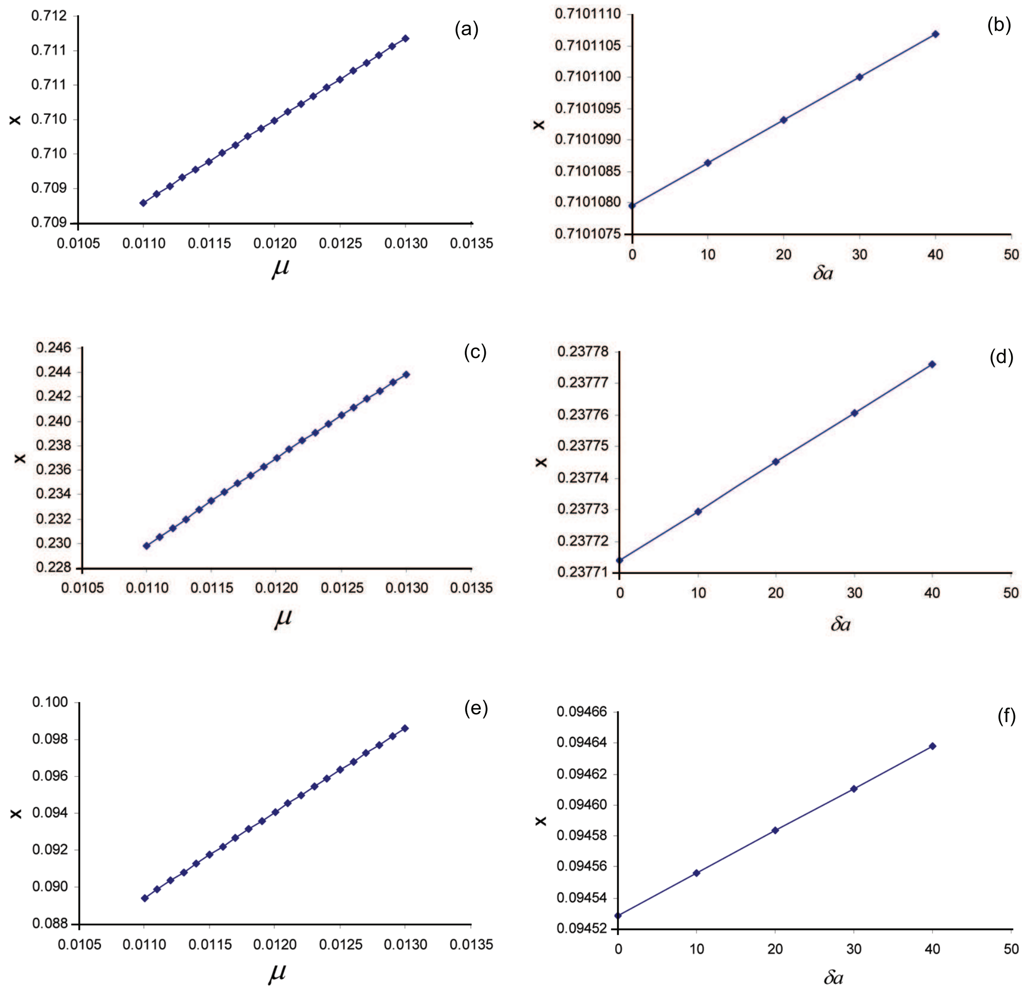

Figure 3. (a), (b) Libration point L_3 for ($\lambda = -2$); (c), (d) Libration point L_3 for ($\lambda = 0$); (e), (f) Libration point L_3 for ($\lambda = 2$).

Table 2. Libration point L_1 [for $(\lambda = -2)$ in the interval $I_1(\mu - 2, \mu - 1)$].

μ	Case-1	Case-2	Case-3	Case-4	Case-5
0.0110	−1.98733955598503	−1.98733938751392	−1.98733921959466	−1.98733905209746	−1.98733888511966
0.0111	−1.98722436532556	−1.98722419685805	−1.98722402894235	−1.98722386144868	−1.98722369447438
0.0112	−1.98710917291981	−1.98710900445569	−1.98710883654336	−1.98710866905303	−1.98710850208205
0.0113	−1.98699397876519	−1.98699381030432	−1.98699364239522	−1.98699347490811	−1.98699330794032
0.0114	−1.98687880744424	−1.98687863898235	−1.98687847107222	−1.98687830358406	−1.98687813661522
0.0115	−1.98676361015634	−1.98676344170369	−1.98676327380272	−1.98676310632367	−1.98676293936386
0.0116	−1.98664841029182	−1.98664824184682	−1.98664807395346	−1.98664790648196	−1.98664773952965
0.0117	−1.98653320807401	−1.98653303963536	−1.98653287174831	−1.98653270428308	−1.98653253733701
0.0118	−1.98641800368351	−1.98641783525015	−1.98641766736836	−1.98641749990836	−1.98641733296749
0.0119	−1.98630279726483	−1.98630262883591	−1.98630246095853	−1.98630229350292	−1.98630212656641
0.0120	−1.98618758893222	−1.98618742050705	−1.98618725263340	−1.98618708518152	−1.98618691824872
0.0121	−1.98607237877469	−1.98607221035273	−1.98607204248229	−1.98607187503360	−1.98607170810398
0.0122	−1.98595716686033	−1.98595699844118	−1.98595683057353	−1.98595666312763	−1.98595649620079
0.0123	−1.98584195324009	−1.98584178482343	−1.98584161695827	−1.98584144951485	−1.98584128259049
0.0124	−1.98572673795087	−1.98572656953648	−1.98572640167358	−1.98572623423243	−1.98572606731032
0.0125	−1.98561152101824	−1.98561135260596	−1.98561118474517	−1.98561101730613	−1.98561085038613
0.0126	−1.98549630245865	−1.98549613404839	−1.98549596618962	−1.98549579875258	−1.98549563183459
0.0127	−1.98538108228137	−1.98538091387307	−1.98538074601625	−1.98538057858117	−1.98538041166514
0.0128	−1.98526586048998	−1.98526569208363	−1.98526552422875	−1.98526535679561	−1.98526518988151
0.0129	−1.98515063708373	−1.98515046867933	−1.98515030082641	−1.98515013339522	−1.98514996648306
0.0130	−1.98503541205859	−1.98503524365618	−1.98503507580524	−1.98503490837603	−1.98503474146585

Table 3. Libration point L_2 [for $(\lambda = -2)$ in the interval $I_2(\mu - 1, 0)$].

μ	Case-1	Case-2	Case-3	Case-4	Case-5
0.0110	−0.0576388436871009	−0.057622464405064	−0.057606128807769	−0.057589824332701	−0.057573560528727
0.0111	−0.0578279114233907	−0.057811705593608	−0.057795543131047	−0.057779411612919	−0.057763320478908
0.0112	−0.0580155955187059	−0.057999560096209	−0.057983567731337	−0.057967606124141	−0.057951684622443
0.0113	−0.058201912744537	−0.058186044758916	−0.058170219519914	−0.058154424858914	−0.058138670023905
0.0114	−0.0583868795658855	−0.058371176122474	−0.058355515116783	−0.058339884505500	−0.058324293438727
0.0115	−0.058570512178986	−0.058554970445717	−0.058539470844882	−0.058524001457454	−0.058508571337741
0.0116	−0.0587528264753185	−0.058737443694525	−0.058722102740365	−0.058706791818455	−0.058691519886266
0.0117	−0.0589338380951298	−0.058918611570984	−0.058903426569139	−0.058888271417990	−0.058873154983108
0.0118	−0.059113562384739	−0.059098489481107	−0.059083457804816	−0.059068455798312	−0.059053492231868
0.0119	−0.0592920144401464	−0.059277092589737	−0.059262211670418	−0.059247360235561	−0.059232546976341
0.0120	−0.0594692090871966	−0.059454435777603	−0.059439703106692	−0.059424999739605	−0.059410334277850
0.0121	−0.0596451609054156	−0.059630533687705	−0.059615946813914	−0.059601389063393	−0.059586868952412
0.0122	−0.0598198842233629	−0.059805400699491	−0.059790957229601	−0.059776542705139	−0.059762165553284
0.0123	−0.0599933931218642	−0.059979050951541	−0.059964748548322	−0.059950474907703	−0.059936238377923
0.0124	−0.060165701445542	−0.060151498343491	−0.060137334719536	−0.060123199679546	−0.060109101490193
0.0125	−0.0603368228053593	−0.060322756531304	−0.060308729455090	−0.060294730781352	−0.060280768699856
0.0126	−0.060506770582367	−0.060492838953516	−0.060478946238159	−0.060465081753523	−0.060451253599143
0.0127	−0.0606755579299764	−0.060661758805687	−0.060647998321034	−0.060634265884503	−0.060620569528012
0.0128	−0.0608431977866464	−0.060829529080548	−0.060815898734995	−0.060802296264249	−0.060788729620476
0.0129	−0.0610097028684953	−0.060996162534928	−0.060982660291898	−0.060969185745117	−0.060955746775450
0.0130	−0.061175085688057	−0.061161671730701	−0.061148295591568	−0.061134946973014	−0.061121633684115

Table 4. Libration point L_2 [for $(\lambda = 0)$ in the interval $I_2(\mu - 1, 0)$].

μ	Case-1	Case-2	Case-3	Case-4	Case-5
0.0110	−0.215088319724763	−0.215073934432473	−0.215059592750509	−0.215045283621240	−0.215031015387973
0.0111	−0.215695137437477	−0.215680889872576	−0.215666685544302	−0.215652513501371	−0.215638382006576
0.0112	−0.216297922583526	−0.216283810249800	−0.216269740785402	−0.216255703342555	−0.216241706105160
0.0113	−0.216896735278186	−0.216882755746563	−0.216868818722520	−0.216854913459900	−0.216841048065783
0.0114	−0.217491634215422	−0.217477785121153	−0.217463978178406	−0.217450202740965	−0.217436466839388
0.0115	−0.218082676715420	−0.218068955756277	−0.218055276597924	−0.218041628692074	−0.218028019995366
0.0116	−0.218669918767290	−0.218656323700931	−0.218642770089598	−0.218629247482333	−0.218615763761666
0.0117	−0.219253415072334	−0.219239943714583	−0.219226513471435	−0.219213113986824	−0.219199753071658
0.0118	−0.219833219082683	−0.219819869305217	−0.219806560307286	−0.219793281825940	−0.219780041600803
0.0119	−0.220409383042563	−0.220396152771291	−0.220382962949230	−0.220369803405130	−0.220356681809494
0.0120	−0.220981958022674	−0.220968845235608	−0.220955772572590	−0.220942729952439	−0.220929724976891
0.0121	−0.221550993959072	−0.221537996684964	−0.221525039214524	−0.221512111555002	−0.221499221240964
0.0122	−0.222116539685359	−0.222103656002156	−0.222090811806668	−0.222077997193376	−0.222065219630471
0.0123	−0.222678642966629	−0.222665870999201	−0.222653138208206	−0.222640434773674	−0.222627768099070
0.0124	−0.223237350531934	−0.223224688451110	−0.223212065239869	−0.223199471162493	−0.223186913558545
0.0125	−0.223792708103459	−0.223780154124307	−0.223767638712202	−0.223755152214377	−0.223742701907705
0.0126	−0.224344760428169	−0.224332312808414	−0.224319903457832	−0.224307522805050	−0.224295178064710
0.0127	−0.224893551303999	−0.224881208343278	−0.224868903357631	−0.224856626856177	−0.224844385992783
0.0128	−0.225439123609057	−0.225426883646961	−0.225414681370105	−0.225402507366772	−0.225390368730755
0.0129	−0.225981519326805	−0.225969380741824	−0.225957279556551	−0.225945206436527	−0.225933168417210
0.0130	−0.226520779572307	−0.226508740780982	−0.226496739107361	−0.226484765293914	−0.226472826317813

Table 5. Libration point L_3 [for $(\lambda = -2)$ in the interval $I_3(\mu, \mu + 1)$].

μ	Case-1	Case-2	Case-3	Case-4	Case-5
0.0110	0.708797211195541	0.708797898935924	0.708798584419654	0.708799268176615	0.708799949809468
0.0111	0.708916419398679	0.708917107046109	0.708917792437191	0.708918476101739	0.708919157642466
0.0112	0.709035617548007	0.709036305102565	0.709036990401080	0.709037673973295	0.709038355421978
0.0113	0.709154805652533	0.709155493114299	0.709156178320329	0.709156861800292	0.709157543157011
0.0114	0.709273983721253	0.709274671090309	0.709275356203934	0.709276039591726	0.709276720856561
0.0115	0.709393151763151	0.709393839039579	0.709394524060880	0.709395207356581	0.709395888529613
0.0116	0.709512309787201	0.709512996971081	0.709513681900138	0.709514365103829	0.709515046185137
0.0117	0.709631457802362	0.709632144893774	0.709632829730669	0.709633512842431	0.709634193832095
0.0118	0.709750595817583	0.709751282816609	0.709751967561421	0.709752650581334	0.709753331479435
0.0119	0.709869723841801	0.709870410748521	0.709871095401331	0.709871778329474	0.709872459136091
0.0120	0.709988841883939	0.709989528698434	0.709990213259323	0.709990896095776	0.709991576810990
0.0121	0.710107949952912	0.710108636675261	0.710109321144308	0.710110003889152	0.710110684513042
0.0122	0.710227048057619	0.710227734687904	0.710228419065189	0.710229101718503	0.710229782251149
0.0123	0.710346136206951	0.710346822745251	0.710347507030854	0.710348189592719	0.710348870034199
0.0124	0.710465214409783	0.710465900856179	0.710466585050181	0.710467267520674	0.710467947871069
0.0125	0.710584282674983	0.710584969029554	0.710585653132033	0.710586335511236	0.710587015770624
0.0126	0.710703341011402	0.710704027274229	0.710704711285265	0.710705393573256	0.710706073741718
0.0127	0.710822389427884	0.710823075599045	0.710823759518718	0.710824441715578	0.710825121793191
0.0128	0.710941427933258	0.710942114012834	0.710942797841223	0.710943479947030	0.710944159933873
0.0129	0.711060456536343	0.711061142524414	0.711061826261598	0.711062508276430	0.711063188172584
0.0130	0.711179475245946	0.71118016114259	0.711180844788649	0.711181526712586	0.711182206518128

Table 6. Libration point L_3 [for $(\lambda = 0)$ in the interval $I_3(\mu, \mu + 1)$].

μ	Case-1	Case-2	Case-3	Case-4	Case-5
0.0110	0.229791130298892	0.229808292877804	0.229825393743244	0.229842446165620	0.229859440285104
0.0111	0.230530602897184	0.230547619507753	0.230564574991036	0.230581482503994	0.230598332269976
0.0112	0.231266057436811	0.231282930659792	0.231299743330120	0.231316508493325	0.231333216454479
0.0113	0.231997553596840	0.232014285944788	0.232030958303501	0.232047583609043	0.232064152246790
0.0114	0.232725149651878	0.232741743571534	0.232758278054485	0.232774765929241	0.232791197660099
0.0115	0.233448902517115	0.233465360391690	0.233481759371513	0.233498112179406	0.233514409357241
0.0116	0.234168867791551	0.234185191942907	0.234201457731193	0.234217677775353	0.234233842693526
0.0117	0.234885099799479	0.234901292490223	0.234917427339597	0.234933516864435	0.234949551757862
0.0118	0.235597651630324	0.235613715065813	0.235629721171926	0.235645682365115	0.235661589412240
0.0119	0.236306575176916	0.236322511507170	0.236338391010610	0.236354226004982	0.236370007329669
0.0120	0.237011921172251	0.237027732493797	0.237043487481918	0.237059198357289	0.237074856030630
0.0121	0.237713739224843	0.237729427582478	0.237745060091150	0.237760648876063	0.237776184918118
0.0122	0.238412077852705	0.238427645241183	0.238443157256480	0.238458625929882	0.238474042311342
0.0123	0.239106984516038	0.239122432881699	0.239137826341508	0.239153176834354	0.239168475478140
0.0124	0.239798505648683	0.239813836891004	0.239829113686576	0.239844347883374	0.239859530666174
0.0125	0.240486686688391	0.240501902661486	0.240517064638923	0.240532184379212	0.240547253132951
0.0126	0.241171572105966	0.241186674620019	0.241201723581704	0.241216730661476	0.241231687174733
0.0127	0.241853205433336	0.241868196255979	0.241883133961947	0.241898030135015	0.241912876154379
0.0128	0.242531629290583	0.242546510148228	0.242561338317483	0.242576125296793	0.242590862528175
0.0129	0.243206885412008	0.243221657991115	0.243236378302900	0.243251057761794	0.243265687871676
0.0130	0.243879014671239	0.243893680619544	0.24390829471456	0.243922868287989	0.243937392904638

Table 7. Libration point L_3 [for $(\lambda = 2)$ in the interval $I_3(\mu, \mu + 1)$].

μ	Case-1	Case-2	Case-3	Case-4	Case-5
0.0110	0.089385449756548	0.089415412448601	0.089445240466530	0.089474957303476	0.089504546112360
0.0111	0.089860274476503	0.089889994878073	0.089919582149091	0.089949059586165	0.089978410471230
0.0112	0.090333585108593	0.090363067404525	0.090392418077686	0.090421660231713	0.090450777275508
0.0113	0.090805401793419	0.090834650059348	0.090863768175697	0.090892779056593	0.090921666235904
0.0114	0.091275744226323	0.091304762432463	0.091333651928770	0.091362435443230	0.091391096632721
0.0115	0.091744631671031	0.091773423685800	0.091802088398139	0.091830648353169	0.091859087328866
0.0116	0.092212082972773	0.092240652566263	0.092269096233435	0.092297436339732	0.092325656782358
0.0117	0.092678116570884	0.092706467418187	0.092734693684997	0.092762817560187	0.092790823058363
0.0118	0.093142750510934	0.093170886195320	0.093198898615725	0.093226809787466	0.093254603840768
0.0119	0.093606002456382	0.093633926472355	0.093661728512469	0.093689430421428	0.093717016443328
0.0120	0.094067889699802	0.094095605456025	0.094123200497009	0.094150696499711	0.094178077820393
0.0121	0.094528429173684	0.094555939995780	0.094583331336613	0.094610624708179	0.094637804577238
0.0122	0.094987637460831	0.095014946594080	0.095042137454210	0.095069231390984	0.095096212980021
0.0123	0.095445530804381	0.095472641416298	0.095499634938199	0.095526532560264	0.095553318965377
0.0124	0.095902125117454	0.095929040300269	0.095955839551888	0.095982543905490	0.096009138149659
0.0125	0.096357435992456	0.096384158765498	0.096410766742605	0.096437280802475	0.096463685837863
0.0126	0.096811478710047	0.096838012022029	0.096864431650475	0.096890758322064	0.096916977032225
0.0127	0.097264268247786	0.097290614979004	0.097316849116897	0.097342991238523	0.097369026440517
0.0128	0.097715819288475	0.097741982252923	0.097768033692712	0.097793994037622	0.097819848484061
0.0129	0.098166146228209	0.098192128175606	0.098217999646091	0.098243780924448	0.098269457305453
0.0130	0.098615263184143	0.098641066801892	0.098666760970153	0.098692365830945	0.098717866776038

Table 8. Non-collinear libration points when $\mu = 0.0121$ and $\lambda = -3$.

Parameter		Case-1	Case-2	Case-3	Case-4	Case-5
L4,5	x	0.076871276	0.0768713663	0.0768714562	0.0768715459	0.0768716353
	y	±0.3998387215	±0.399838833	±0.3998389438	±0.3998390535	±0.3998391623
L6,7	x	0.8725696423	0.8725698901	0.8725701393	0.8725703893	0.8725706369
	y	±0.0000059524	±0.0000021374	±0.0000008284	±0.0000008751	±0.0000005854

Table 9. The characteristic roots of $L_i, i = 1, \ldots, 5$ for ($\mu = 0.0121$).

Parameter		Case-1	Case-2	Case-3	Case-4	Case-5
$L_1(\lambda=-2)$	$\Lambda_{1,2}(\pm)$	2.22224809266244	2.22224903363355	2.22224997152378	2.22225090705808	2.22225183969274
	$\Lambda_{3,4}(\pm)$	1.66318397388982i	1.66318465715318i	1.66318533817567i	1.6631860174743i	1.66318669465992i
$L_2(\lambda=-2)$	$\Lambda_{1,2}(\pm)$	0.544034083608638i	0.54390669383424i	0.543779647701544i	0.543652873999452i	0.543526431200208i
	$\Lambda_{3,4}(\pm)$	198.00481600788i	198.113470879816i	198.221906950276i	198.330207090033i	198.438307495963i
$L_2(\lambda=0)$	$\Lambda_{1,2}(\pm)$	0.368741090684038i	0.368711582984698i	0.36868216477793i	0.368652815058834i	0.368623549944599i
	$\Lambda_{3,4}(\pm)$	19.7837379442687i	19.786046407193i	19.7883481973936i	19.7906450828423i	19.7929357242807i
$L_3(\lambda=-2)$	$\Lambda_{1,2}(\pm)$	1.26640549436458	1.26640775331612	1.26641000489232	1.26641225092508	1.26641449005445
	$\Lambda_{3,4}(\pm)$	0.966727227026708i	0.966728088442753i	0.966728947529821i	0.966729806178976i	0.966730663156168i
$L_3(\lambda=0)$	$\Lambda_{1,2}(\pm)$	0.43189498565155	0.431942510955709	0.43198986572753	0.432037088810825	0.432084151651992
	$\Lambda_{3,4}(\pm)$	18.1149561803198i	18.1125151432477i	18.1100832846405i	18.1076587191353i	18.1052428414995i
$L_3(\lambda=2)$	$\Lambda_{1,2}(\pm)$	0.563899510533276	0.564068741005145	0.564237115478347	0.564404788543882	0.564571654049788
	$\Lambda_{3,4}(\pm)$	147.318234346153i	147.2367408581i	147.155672299404i	147.074964116808i	146.994661318818i
$L_{4,5}(\lambda=-3)$	$\Lambda_{1,2}(\pm)$	1.51156415865594	1.5115629797817	1.51156180368639	1.51156062450909	1.51155944636281
	$\Lambda_{3,4}(\pm)$	2.66627611160177i	2.66627665088885i	2.66627719089646i	2.66627773251176i	2.66627827474513i
$L_{4,5}(\lambda=-3)$	$\Lambda_{1,2}$	0.24205 ±0.45849i	0.24204 ±0.45849i	0.24203 ±0.45849i	0.24202 ±0.45849i	0.24201 ±0.45849i
	$\Lambda_{3,4}$	−0.24205 ±0.4585i	−0.24204 ±0.4585i	−0.24203 ±0.4585i	−0.24202±0.4585i	−0.24201±0.4585i

5. Conclusion

We observe that the existence and position of equilibrium points ($L_i, i = 1, \ldots, 7$) depends on λ (magnetic moments ratio), μ (mass ratio), and σ (parameter due to triaxial body).

Existence of equilibrium points depend upon the λ:

λ	L_1	L_2	L_3	$L_{4,5}$	$L_{6,7}$	Total
≥ 3	Does not exist	Does not exist	Does not exist	Does not exist	Does not exist	0
2	Does not exist	Does not exist	exists	Does not exist	Does not exist	1
0	Does not exist	exists	exists	Does not exist	Does not exist	2
-2	exists	exists	exists	Does not exist	Does not exist	3
≤ -3	exists	exists	exists	exists	exists	7

We observe that there exists upto seven equilibrium points.

The effect of changing the mass ratio (mu) and the change in semi-major axis, (delta a) moves the equilibrium points towards or away from the centre of mass. More precisely, as (mu) increases L_1 moves towards the centre of mass whereas L_2 and L_3 move away from the centre of mass. On the other hand a small increase in (delta a) moves L_1 and L_2 towards the centre of mass and L_3 away from the centre of mass.

We have also found that only L_2 is stable for some combinations of μ and λ. Taking the bigger primary as triaxial, the magnetic binary problem become more realistic than that with spherical primaries. This is because the shape of primaries influences the motion of two body system.

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