Imaging in Radio Astronomy

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Fourier transform and imaging

$$A(l,m).I(l,m) = \int_{-\alpha}^{\alpha} \int_{-\alpha}^{\alpha} V(u,v) e^{2\pi i(\mathrm{ul}+\mathrm{vm})} \mathrm{dudv} \quad \text{---}(1)$$

Holds good if $\Delta\nu.\Delta\tau_g \ll 1$ and
$w.(n-1) \ll 1$ or $w.(l^2+m^2) \ll 1$.

(1) holds if $V(u,v)$ is a continuous fn.
In practice, it is discrete and uneven.

Rewrite (1) as a DFT relation:

$$I(l,m) = \frac{1}{M} \sum_{k=1}^{M} V(u_k, v_k) e^{2\pi i(u_k l + v_k m)}$$

Requires αN^4 computation

FFT is faster.

Map resolution and pixel size

Highest value of u, v.
F.T. relation.
Resolution $\sim (u^2+v^2)^{-0.5}$.

How small is $\boldsymbol{\Delta l, \Delta m}$?

Nyquist criterion:

$\Delta l < \frac{1}{2.u_{\max}}, \quad \Delta m < \frac{1}{2.v_{\max}}$

$\Delta l, \Delta m$ are pixels (or Cells).

How many pixels in a map ?

Want to cover maximum possible sky area.

Limited by Primary beam size.

$N_l = \frac{\lambda}{D.\Delta l}.$

F.T. and diffraction pattern of a source

Idea of Beam:

$S(u,v) = \sum \delta(u - u_k, v - v_k)$

S -- Sampling function

$I(l,m) = \frac{1}{M} \sum_{k=1}^{M} S.V'(u_k, v_k) e^{2\pi i(u_k l + v_k m)}$

$I_D = F(V^S) = F(S.V')$

Let's consider V' as a continuous visibility function.

$$I_D = F(S) * F(V')$$
$$I_D = B * F(V')$$

F.T. of a 1-D Box is Sinc $\left(\frac{\sin(x)}{x}\right)$.

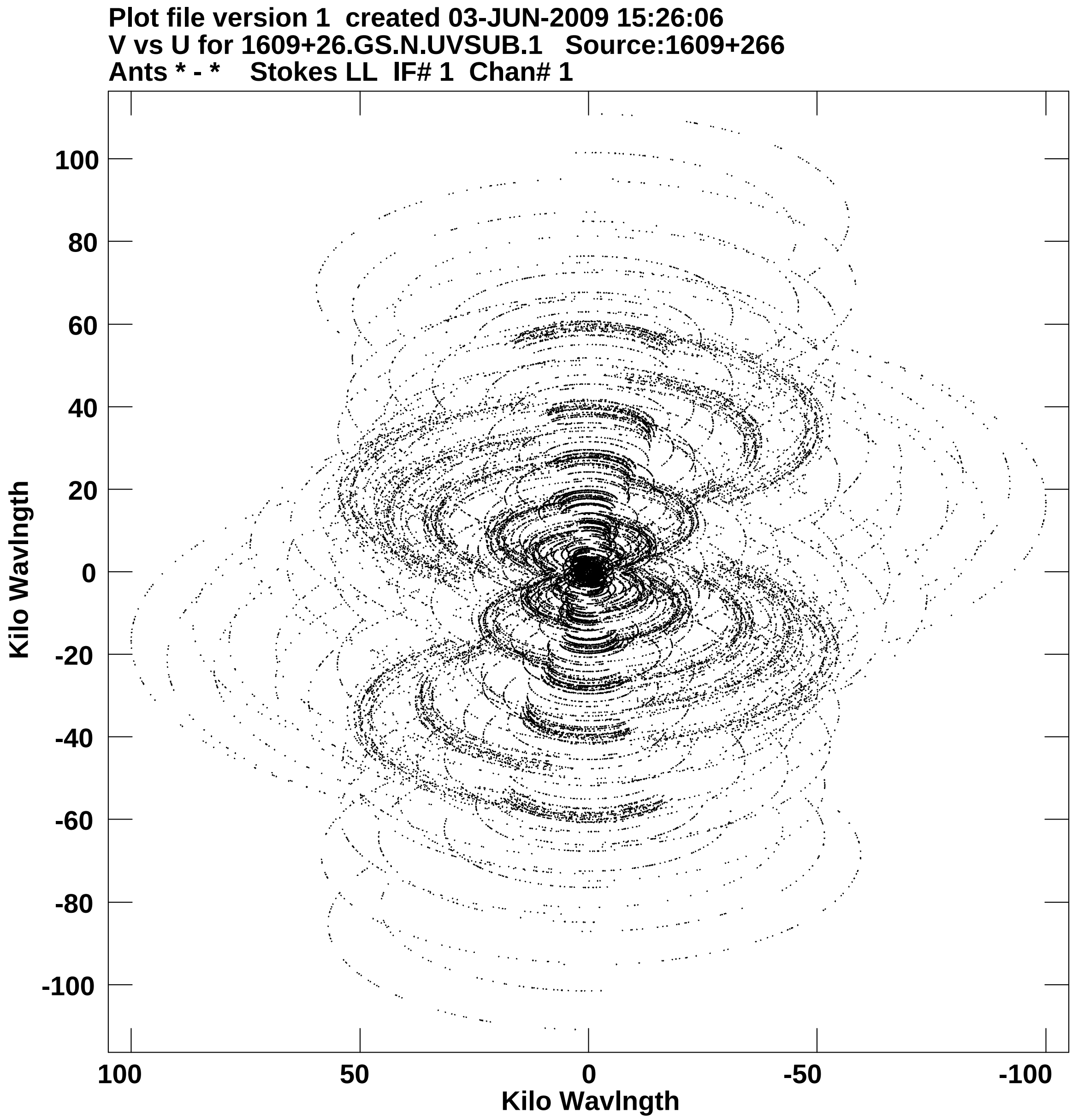

Figure 1. u, v coverage for the source 1609+266 with nearly full synthesis.

Various data Weights

Data weights changes Beam pattern to reduce Diffraction Sidelobes.

Data Tapering (T_k), Density weight (D_k) and reliability weight (R_k):

Tapering: Reduced contribution from edges.

Density weight:
Data from different parts of u, v plane gets uniform weight (uniform weighting).
Density weight=1 (Natural weighting).
Reliability weight:
More noisy data from a few antennas get reduced weight.
$S^W(u,v) = \sum T_k.D_k.R_k.\delta(u-u_k, v-v_k)$.

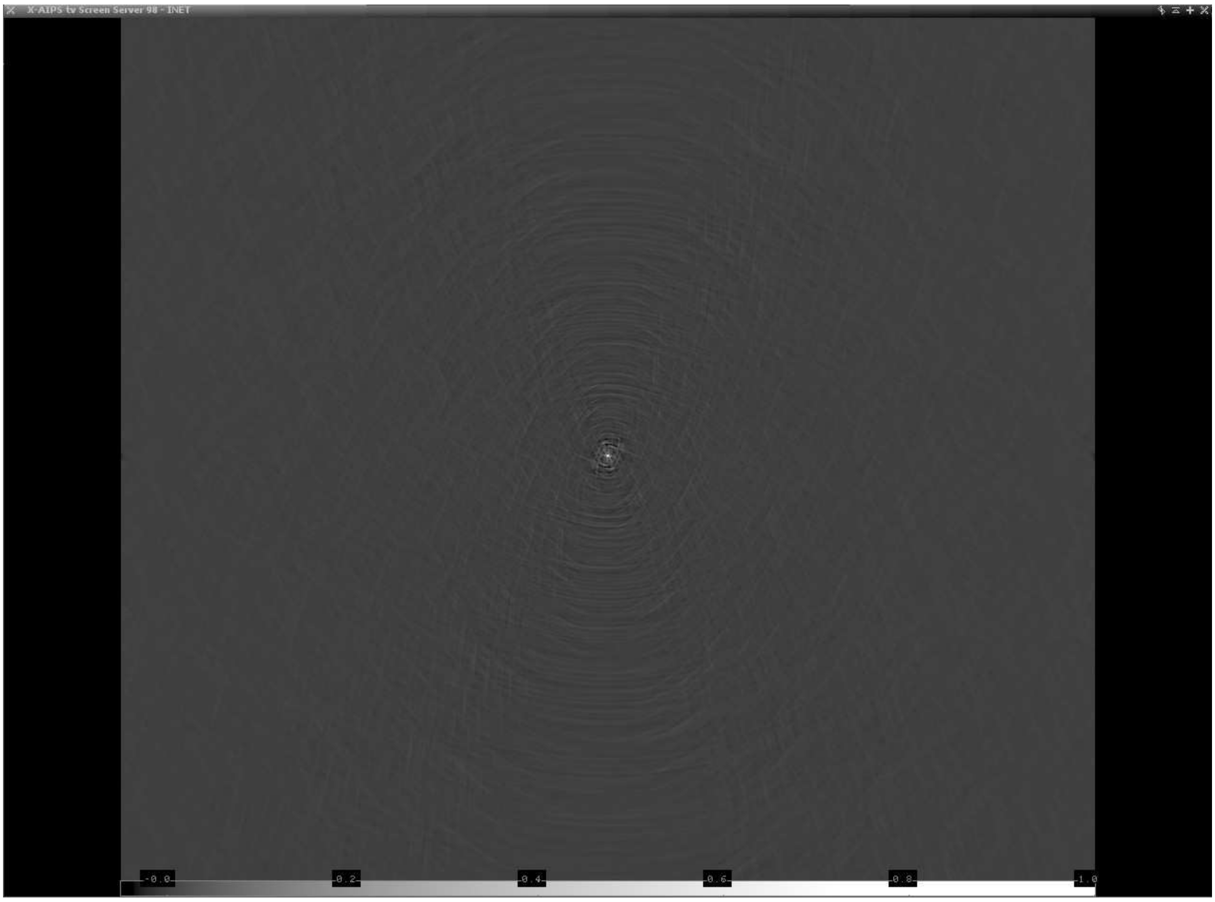

Figure 2. Beam pattern for the full synthesis data of GMRT on 1609+266.

Various assumtions and their effect on the map

Non-coplanar baselines and the 'w' term

$w(l^2+m^2)\not\ll 1$.

Use multiple facets each of which corrects for w term at the facet centre (polyhedron imaging).

Frequency channel averaging and Bandwidth smearing

u, v varies due to finite bandwidth.

Time averaging of data and source smearing

Source $V(u,v)$ changes with 't'.