What all did I say wrongly yesterday ?

It is a good time for you to now bring it up and

for me to try to say it again,

now hopefully correctly.

System performance characterization

System (or Antenna) Gain (antenna temperature per unit flux density) :

power received per pol = ½ S Aeff B = k Tantenna B

Gain : Aeff /2k (usually given in K per Jansky)

(1K per ~2800 sq m effective area)

System Temperature (Tsys = Tantenna + Treceiver + Tground)

System (Tsys) Equivalent Flux Density (SEFD)

Tsys can be calibrated in terms of some routinely available reference, say, Tcal … which in turn is calibrated with known temperature sources

Radiometer Equation

$$\mathbf{T_{rms} = \alpha\, T_{sys} / (\Delta\nu\, t_{int}\, N_p)^{½}}$$

T_{rms} = r.m.s. noise in observation

α ~ inefficiency factor

e.g. $(2)^{1/2}$ since you have to switch

off-source → position switch

off-frequency → frequency switch

T_{sys} = System temperature (includes antenna temperature)

$\Delta\nu$ = bandwidth, frequency range observed

t_{int} = integration time (how long is the exposure?)

N_p = No. of orthogonal polarization states

Johnson-Nyquist Noise

$\langle V \rangle = 0$, but $\langle V^2 \rangle \neq 0$

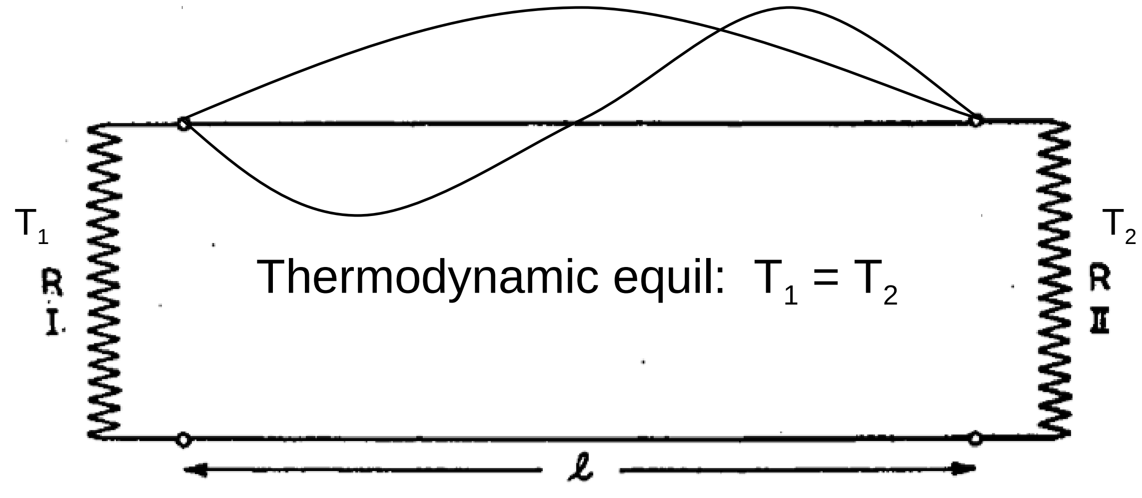


Fig. 3.

"Statistical fluctuations of electric charge in all conductors produce random variations of the potential between the ends of the conductor…producing mean-square voltage" => white noise power, $\langle V^2 \rangle/R$, radiated from resistor at T_R

- Transmission line electric field standing wave modes: $\nu = c/2l, 2c/2l \ldots Nc/2l \ldots$
- # modes (=degree freedom) in $\nu + \Delta\nu$: $\Delta N = 2l\, \Delta\nu / c$
- Therm. Equipartion law: energy/degree of freedom: $\Delta E = h\nu/(e^{h\nu/kT} - 1) \sim kT$ (RJ)
- Energy equivalent on line in $\Delta\nu$: $E = \Delta E\, \Delta N = (kT2l\Delta\nu) / c$
- Transit time of line: $t \sim l / c$
- average power transferred from each R to line in $\Delta\nu \sim E/t$ = $\boxed{P_R = \mathbf{kT_R}\, \Delta\nu}$

Johnson-Nyquist Noise

Thermal noise:

$\langle V^2 \rangle / R$ = 'white noise power'

k_B = 1.27 +/ 0.17 erg/K

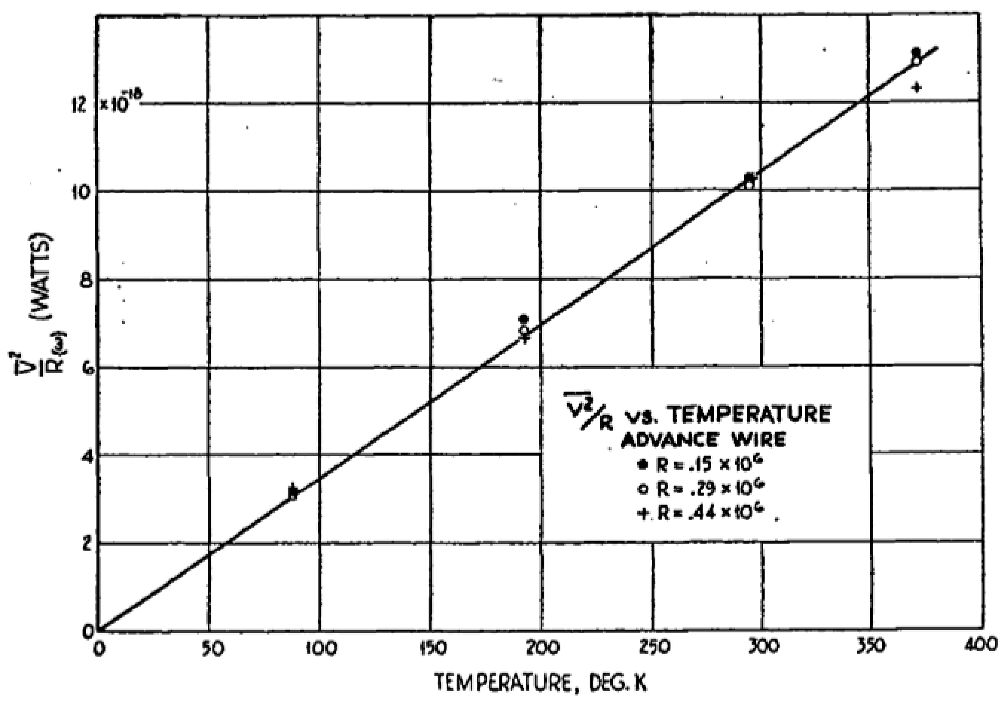


Fig. 6. Apparent power vs. temperature, for Advance wire resistances.

- Noise power is strictly function of T_R, not function of R or material…
- Dickey shows direct analogy with thermal radiation from Black Body
- Nyquist shows direct analogy with thermal motions of molecules in a gas

Interferometric Radiometer Equation

Interferometer pair: $$DT_{lim} = \frac{T_{\mathrm{sys}}}{\sqrt{D\nu\, t}}$$

Antenna temp equation: $\Delta T_A = A_{eff}\ \Delta S_\nu / k$

Sensitivity for single interferometer: $$DS_{lim} = \frac{\mathrm{kT}_{\mathrm{sys}}}{A_{\mathrm{eff}}\sqrt{D\nu\, t}}$$

Finally, for an array, the number of independent measurements at give time = number of pairs of antennas = $N_A(N_A\text{-}1)/2$

$$DS_{lim} = \frac{\mathrm{kT}_{\mathrm{sys}}}{A_{\mathrm{eff}}\sqrt{N_A\left(N_A-1\right)D\nu\, t}}$$

Can be generalized easily to: # polarizations, inhomogeneous arrays (A_i, T_i), digital efficiency terms…

Fun with noise: Wave noise vs. counting statistics

- Received source power $\propto$ telescope area = A_{eff}

- Optical telescopes: $n_s < 1 \Rightarrow$ rms ~ $n_s^{1/2}$

$n_s \propto A_{eff}$ => SNR = signal/rms $\propto (A_{eff})^{1/2}$

- Radio telescopes: $n_s > 1 \Rightarrow$ rms ~ n_s

$n_s \propto T_{sys} = T_{Rx} + T_A + T_{BG} + T_{spill}$

- Faint source: $T_A \ll (T_{Rx} + T_{BG} + T_{spill})$ => rms dictated completely by receiver (independent of A_{eff}) => SNR $\propto A_{eff}$

- Bright source: $T_{sys} \sim T_A \propto A_{eff}$ => rms $\propto A_{eff}$

=> SNR *independent* of A_{eff}

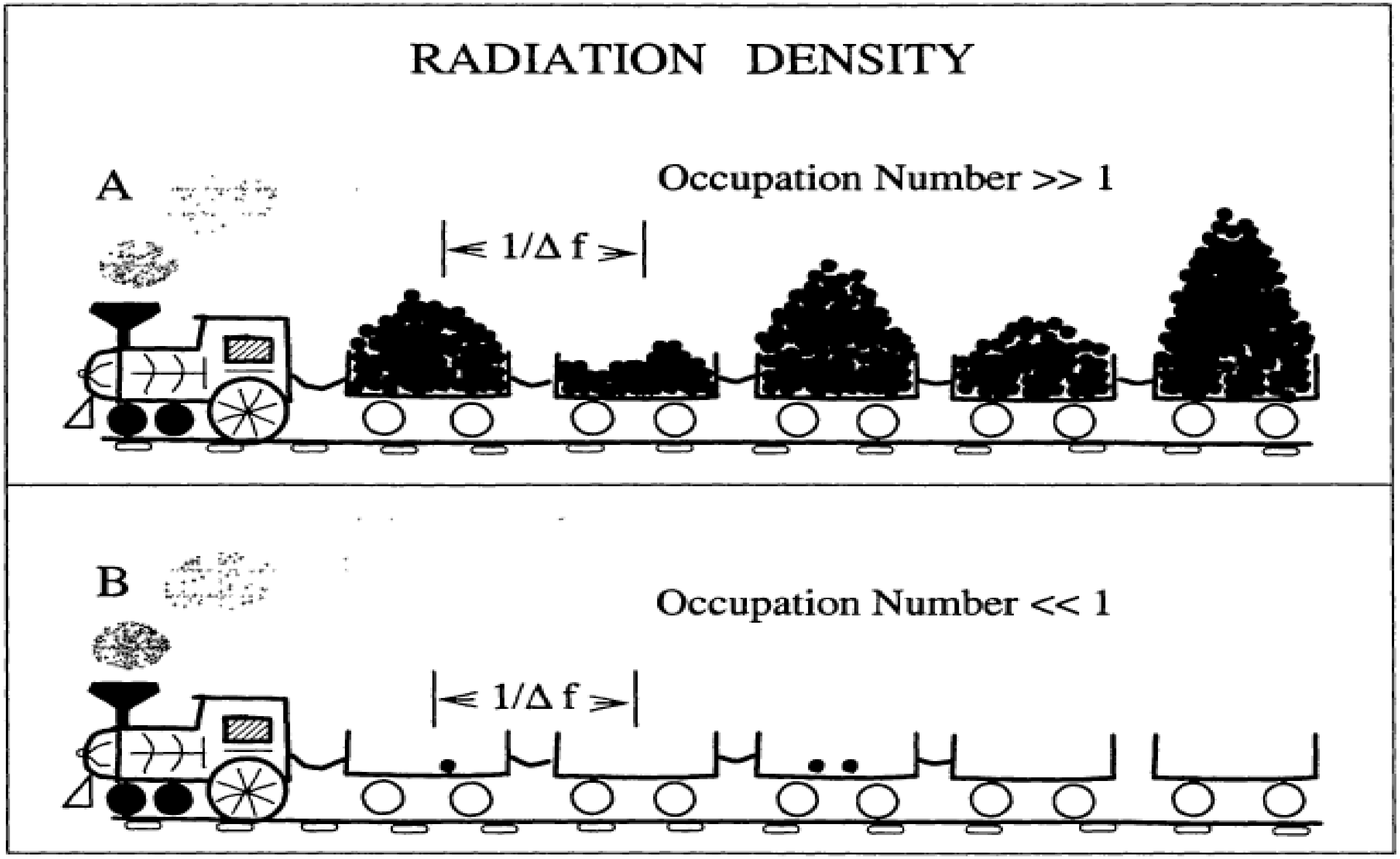


Figure 33–1. Boxcar representation for a stream of radiation. Each boxcar is a sample and corresponds to the reciprocal of the bandwidth, the rate at which new information arrives. A) The high density case where there is an enormous number of photons in each sample and substantial variation from sample to sample. B) The very low density case when the number of photons is minute compared to the number of samples.

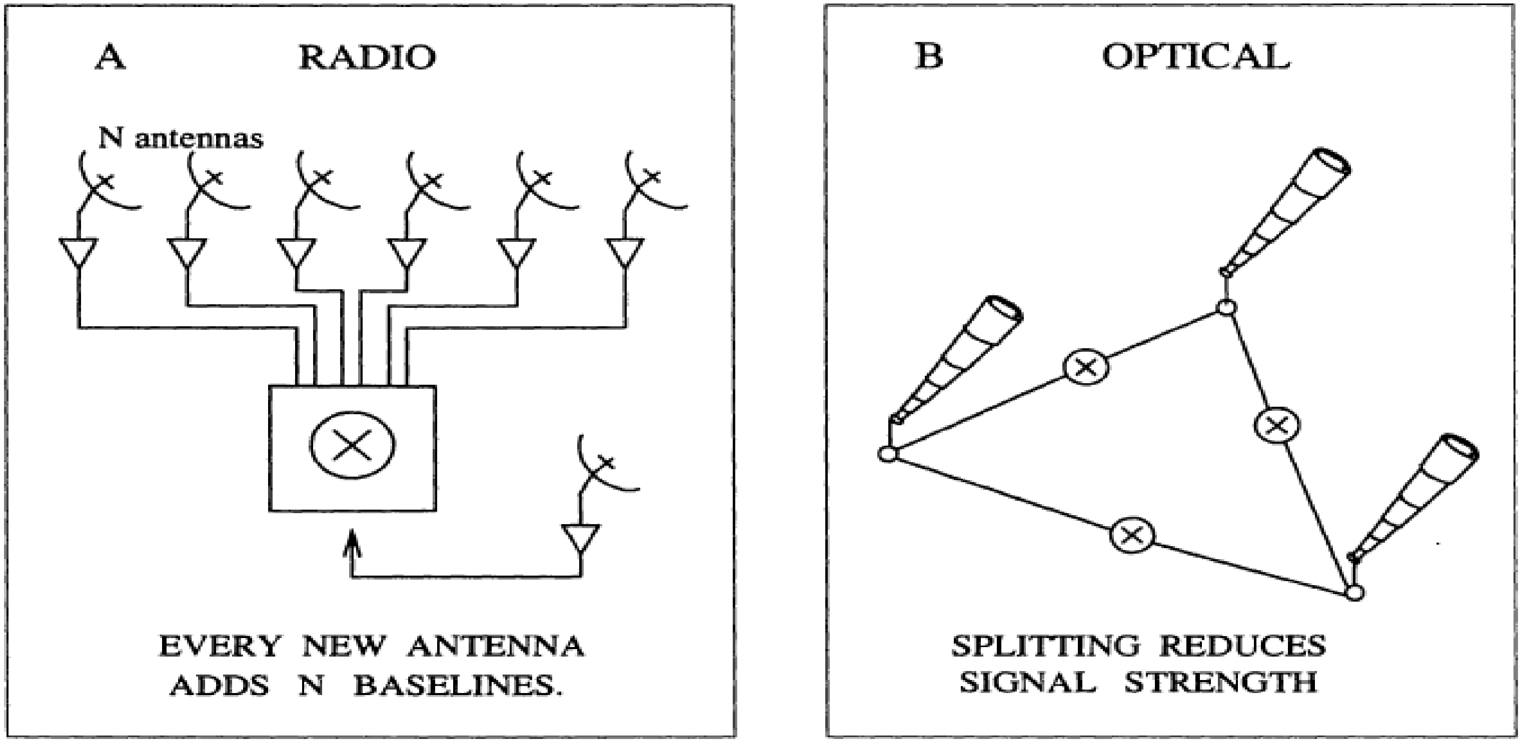
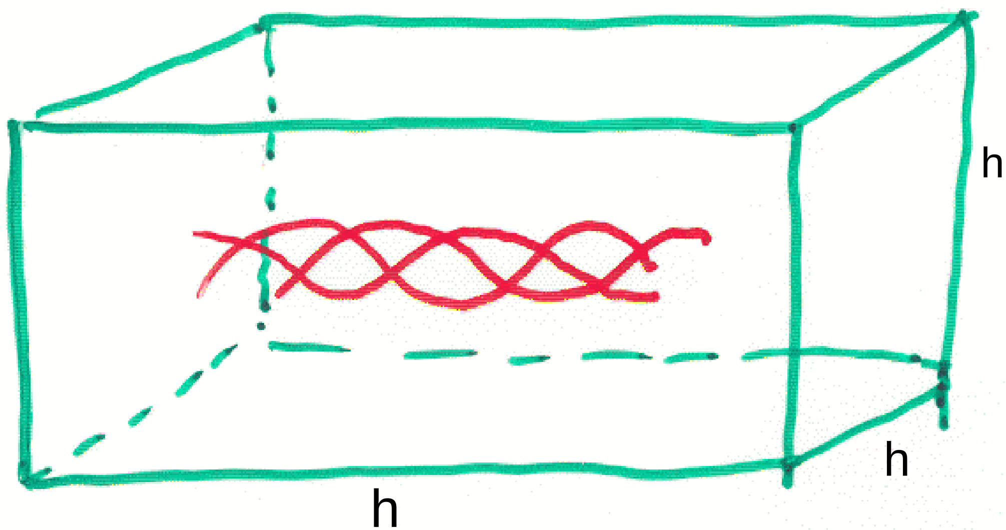


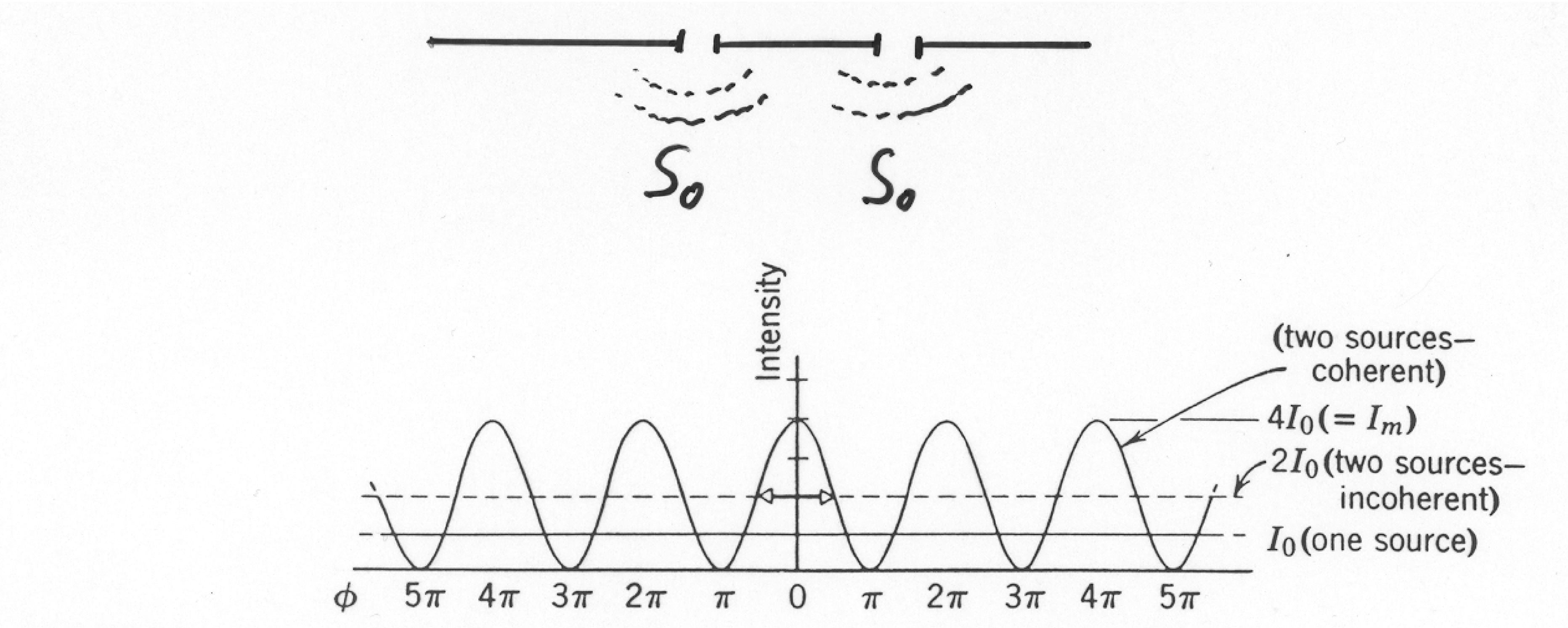
Figure 33–7. Aperture synthesis with and without amplifiers. A) In a radio array, the signal from each antenna is amplified and split N ways to be correlated simultaneously with the signals from all the other antennas. B) In an optical array of Michelson interferometers, splitting the signal reduces its strength and has to be compensated by increased observation to provide the same number of photons per baseline as without splitting.

Origin of wave noise: ‘Bunching of Bosons’ in phase space (time and frequency) allows for interference (ie. coherence).



Bosons can, and will, occupy the exact same phase space if allowed, such that interference (destructive or constructive) will occur. Restricting phase space (ie. narrowing the bandwidth and sampling time) leads to interference within the beam. **This naturally leads to fluctuations that are proportional to intensity (= wave noise).**

Origin of wave noise: coherence -- Young's 2 slit experiment



Single Source: $I \mu V^2 = ©1\text{photon}©$

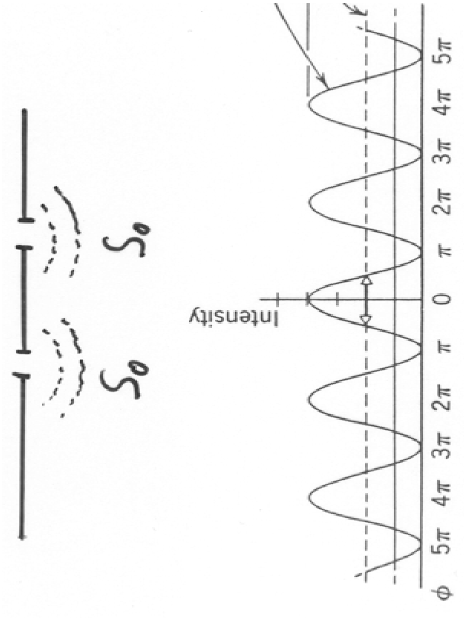
Two incoherentsources: $I \mu 2\left(V^2\right) = ©2\text{photons}©$

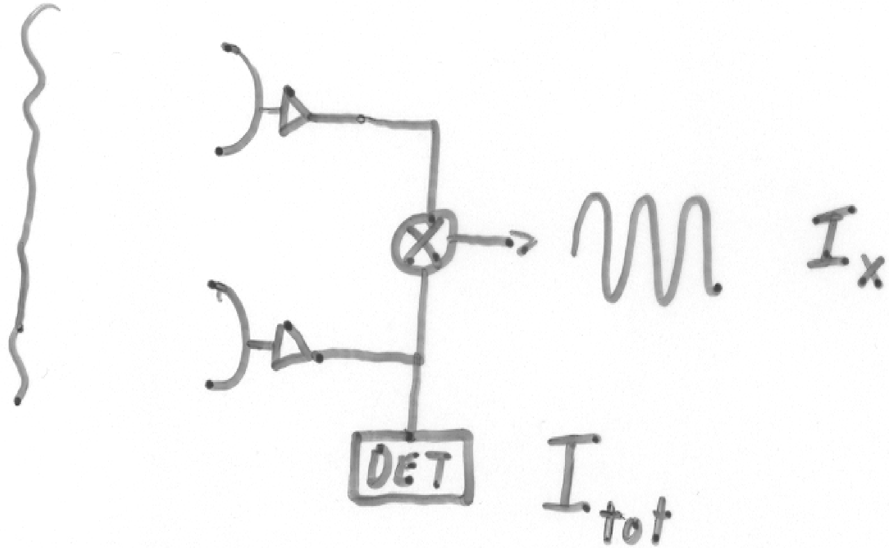
Two coherentsources: $I \mu \left(2\mathrm{V}\right)^2 = ©0\text{to}4\text{photons}\,'$

Wave noise: conclusions

In radio astronomy, the noise statistics are wave noise dominated, ie. rms fluctuations are proportional to the total power (n_s), and not the square root of the power ($n_s^{1/2}$)

Quantum noise and the 2 slit paradox



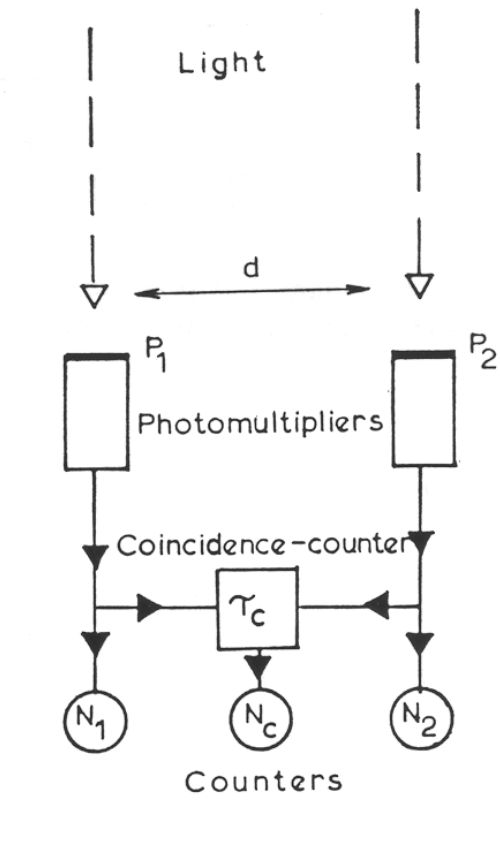


Which slit does the photon enter? With a phase conserving amplifier it seems one could both detect the photon and ‘build-up’ the interference pattern (which we know can’t be correct). But quantum noise dictates that the amplifier introduces 1 photon/mode noise, such that:

$$I_{tot} = 1 \text{ +/- } 1$$

and we still cannot tell which slit the photon came through!

<u>Intensity Interferometry:</u> rectifying signal with square-law detector ('photon counter') destroys phase information. Cross correlation of intensities still results in a finite correlation, proportional to the square of E-field correlation coefficient as measured by a 'normal' interferometer. **Exact same phenomenon as increased correlation for t < 1/Δν in lag-space above,** ie. correlation of the wave noise itself = 'Brown and Twiss effect'



$$\bar{N}_c = \bar{N}_1 \bar{N}_2 2\tau \left[1 + \frac{1}{2}\gamma^2 \right]$$

γ = correlation coefficient

- **Voltages correlate on timescales ~ 1/ν, with correlation coef, γ**

- **Intensities correlate on timescales ~ 1/Δν, with correlation coef, γ^2**

Advantage: timescale = 1/Δν (not 1/ν)

=> insensitive to poor optics, 'seeing'

Disadvantage: No visibility phase information

lower SNR

Polarization measurements

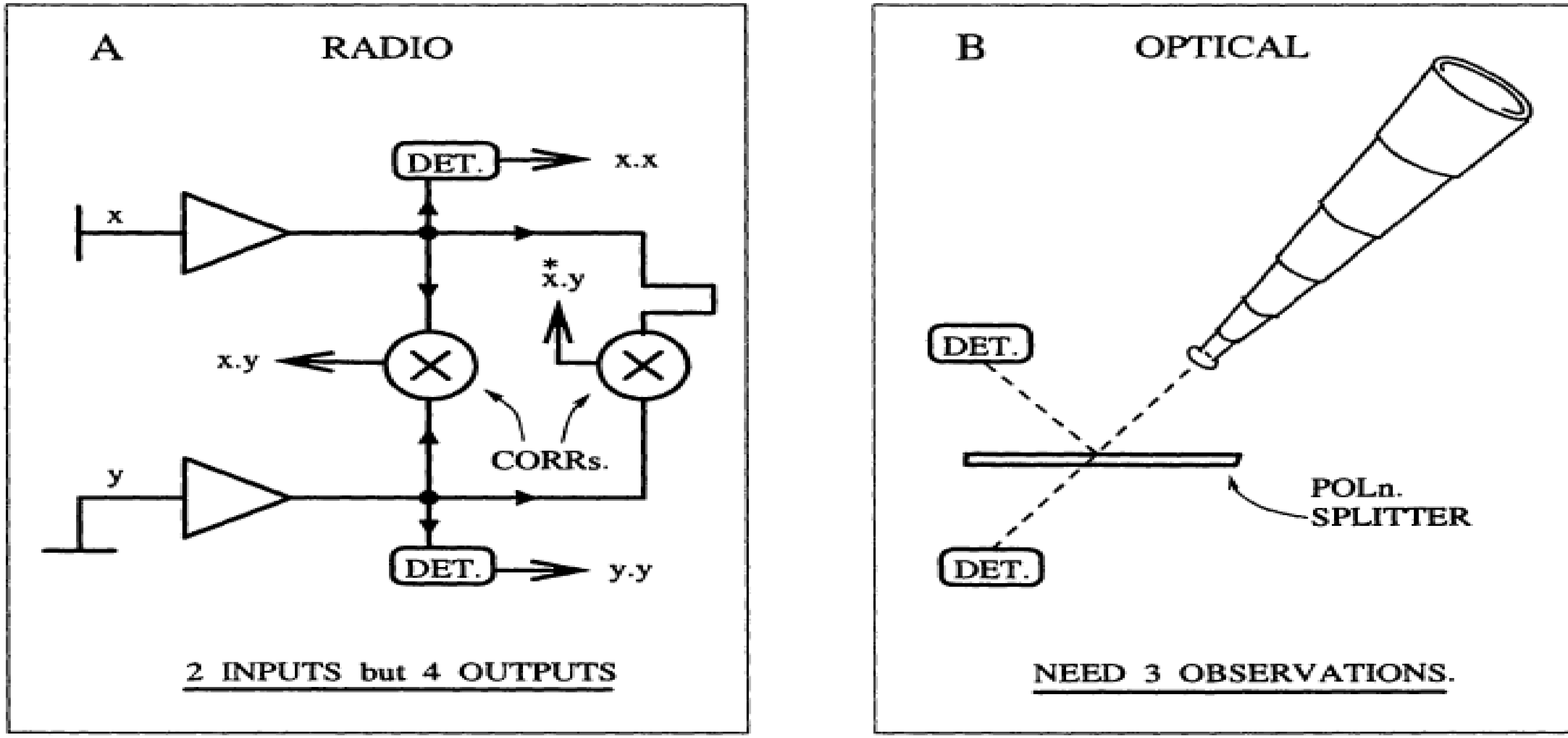


Figure 33–6. The measurement of polarization with and without amplifiers. A) At radio frequencies amplifying and splitting the signal from orthogonally polarized feeds permits the simultaneous determination of all four Stokes parameters. B) An optical telescope would require three observations in sequence for the same determination. Two of the six measurements obtained with different polarization splitters are redundant, but cannot be avoided.

Polarization measurements

- Stokes parameters I,Q,U,V
- For linear-pol inputs :

XX*+YY*; XX*-YY*; real & imag of (XY*)

for circular inputs:

LL*+RR*; imag & real of (LR*); LL*-RR*

For randomly polarized signal: U=Q=V=0

(Poincare sphere: Q,U,V define 3 orthogonal axes)

For each Δt (=1/B), polarization is always 100%

rate of information → bandwidths (two kinds)

- Temporal frequencies (c/lambda)
  inverse of span → limiting time resolution
  inverse of resolution → minimum time-span

- Spatial frequencies (d/lambda)
  inverse of span → limiting angular resolution
  inverse of resolution → field of view

Note: the spatial frequencies depend also on temporal frequencies

Available info v/s extracted info

- The amount of independent information (N) provides the level of confidence in the estimation of desired parameters
- Sqrt(N), for random info
- Level of confidence: judged by asking if the result could have been produced by chance due to random noise, and with what probability ?   (example of a profile, choice of threshold)

Linear systems :Convolutions with impulse responses describe the outputs

- Same as multiplying the signal spectrum by a spectral response of a device or filter

- Looking for some pattern in the signal ?

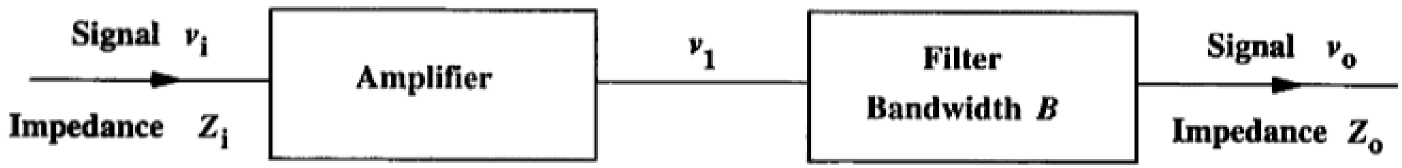
want to check its similarity or correlation with something ?

physical devices can only convolve, but

they can be tricked into performing correlation

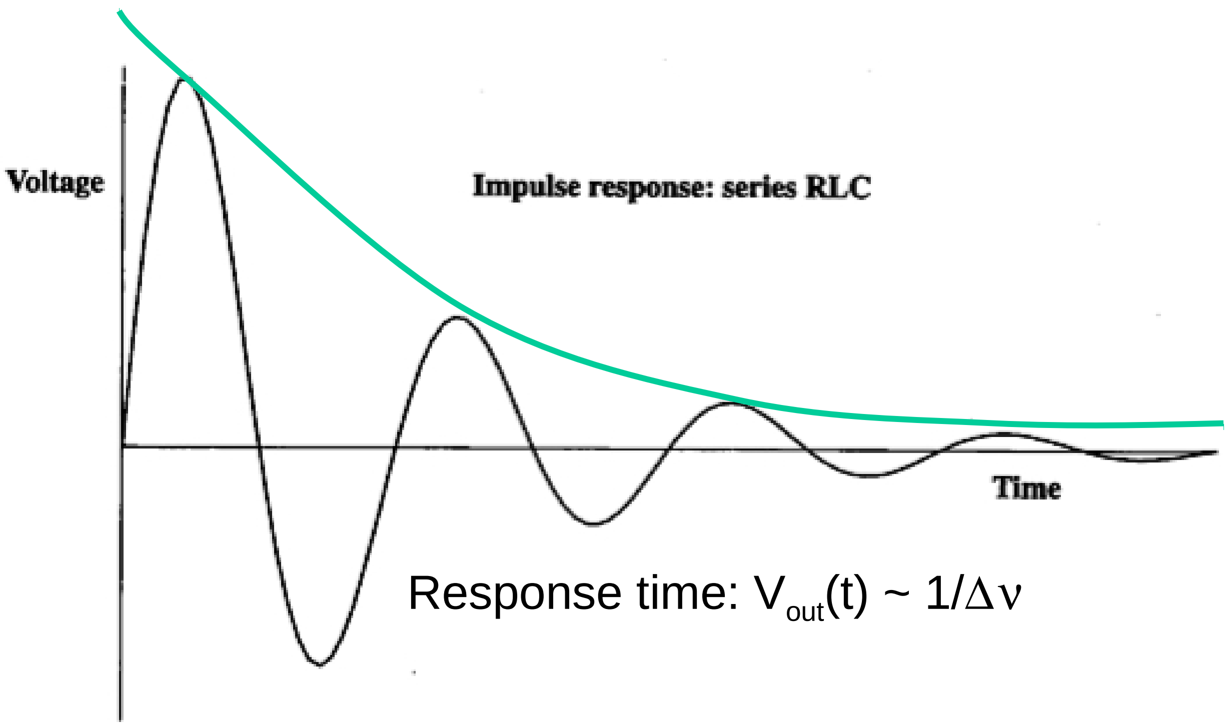
Response time of a bandpass filter

$V_{in}(t) = \delta(t)$



$\Delta\nu$

$V_{out}(t) \sim 1/\Delta\nu$



Response time: $V_{out}(t) \sim 1/\Delta\nu$

Fig. 3.2. Impulse response of an RLC filter.

Response of RLC (tuned) filter of bandwidth $\Delta\nu$ to impulse $V(t) = \delta(t)$: decay time ('ringing') $\sim 1/\Delta\nu$

Matched-filtering

- Optimum detection procedure in presence of random white noise (also Gaussian! why ?).
- We want to exploit all the available signal energy to build up a contrast between signal and noise !

e.g., realizing auto-correlation (at zero delay) !

How to use convolution to perform correlation ?

Even the square-law detection may be thought of as a "matched"-filtering operation.

Effectively, pattern matching/recognision, but in which domain ? which provides best contrast ? (information is not changed across Fourier domains)

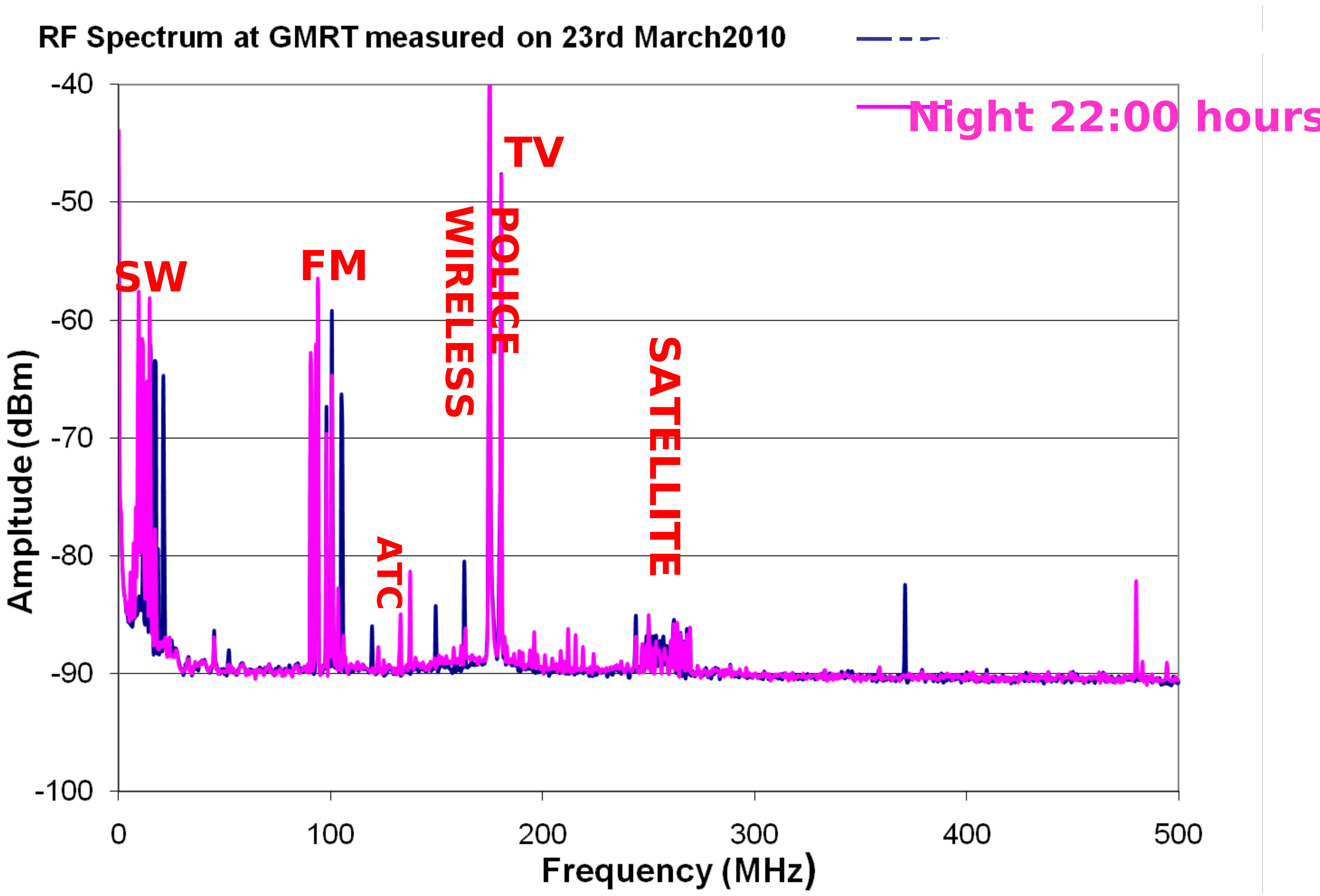
e.g, pulsar search and study techniques, optimum weighting schemes in estimation of average values of quantities, etc.

Maximum power transfer: impedance matching and implications of mismatches

- Reflections v/s coupling
- Interference from reflected signals → standing waves (spatial and spectral)
- Drop in coupling : loss of signal, and efficiency

Similarly, improper or non-“matched” filtering, *even when performed in software*, results in loss due to the corresponding “mis-match”

RF Spectrum at GMRT : 0 – 500 MHz (Praveen)



RFI from communication systems

- Radars
- Radio (FM), TV broadcasts, HAMs+repeaters
- Mobile phones, other wireless networks
- Satellites (telecom satellites; e.g. Iridium)
- Only about a percent of the spectrum is reserved for radio astronomy, and the regulations on other bands, and on their spill-over, are not adequately stringent…… while sometimes RA needs use of the whole spectral range, though passive use and in some locations.

Other, local and often unanticipated, sources:

- Lightning
- Power-line discharges
- Oscillation in TV boosters at homes
- Microwave ovens
- Cordless phone sets
- Fluorescent lights
- Sparking, ignition plugs/cars
- Leakages from labs/RA receivers/computers

Some sources of radio frequency interference (RFI) are inescapable.
Locating telescopes at remote sites may help in minimizing terrestrial interference: e.g. MRO in WA, MeerKat in SA
And/Or seek explicit protection … “Radio quiet zones”
No escape from RFI generated by satellite transmitters, such as those of the Iridium System.

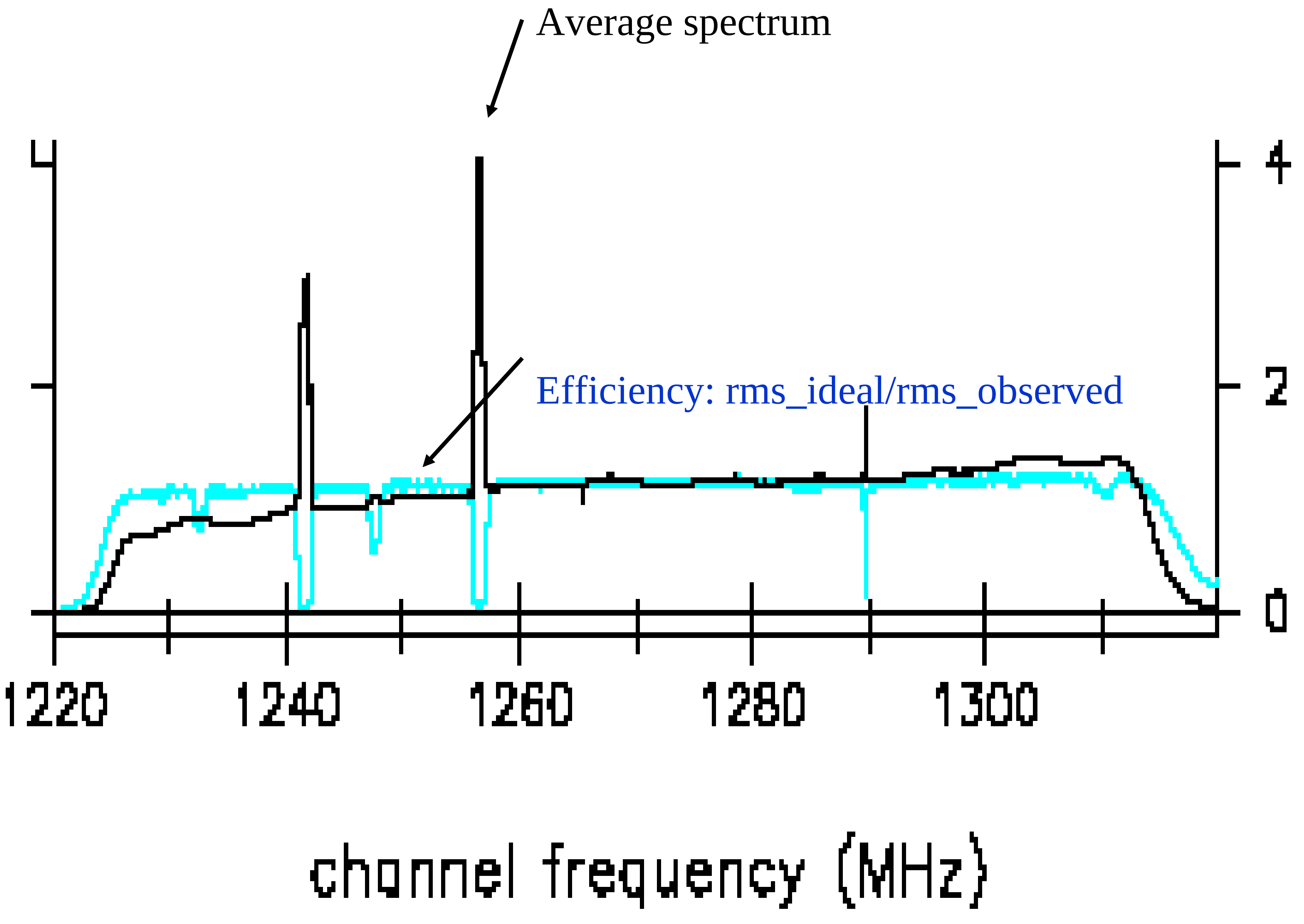
Other truly remote locations ?: Expensive!
The other side of the Moon ?
L2 : Lagrange point ?

Broader definition of RFI: analogous to clutter (as in Radar lingo): anything that contaminates the *desired !*, which we call our signal, even though it is *noise*.

- Radiation of receiver (LNA) noise- reflection/echo → standing wave patterns in/across the spectrum (can also be due to any impedance mismatch in the signal path)
- Sun (for non-solar astronomy; e.g. solar bursts, or even quiet Sun) (Moon ? although a potential calibrator)
- Pulsars or variable sources/transients or “self-noise” from very bright continuous source (for non-transient sky observation; e.g. synthesis imaging)
- HI-emission contamination while studying HI absorption (when reference spectrum is unavailable)

Broader definition of RFI: continued..

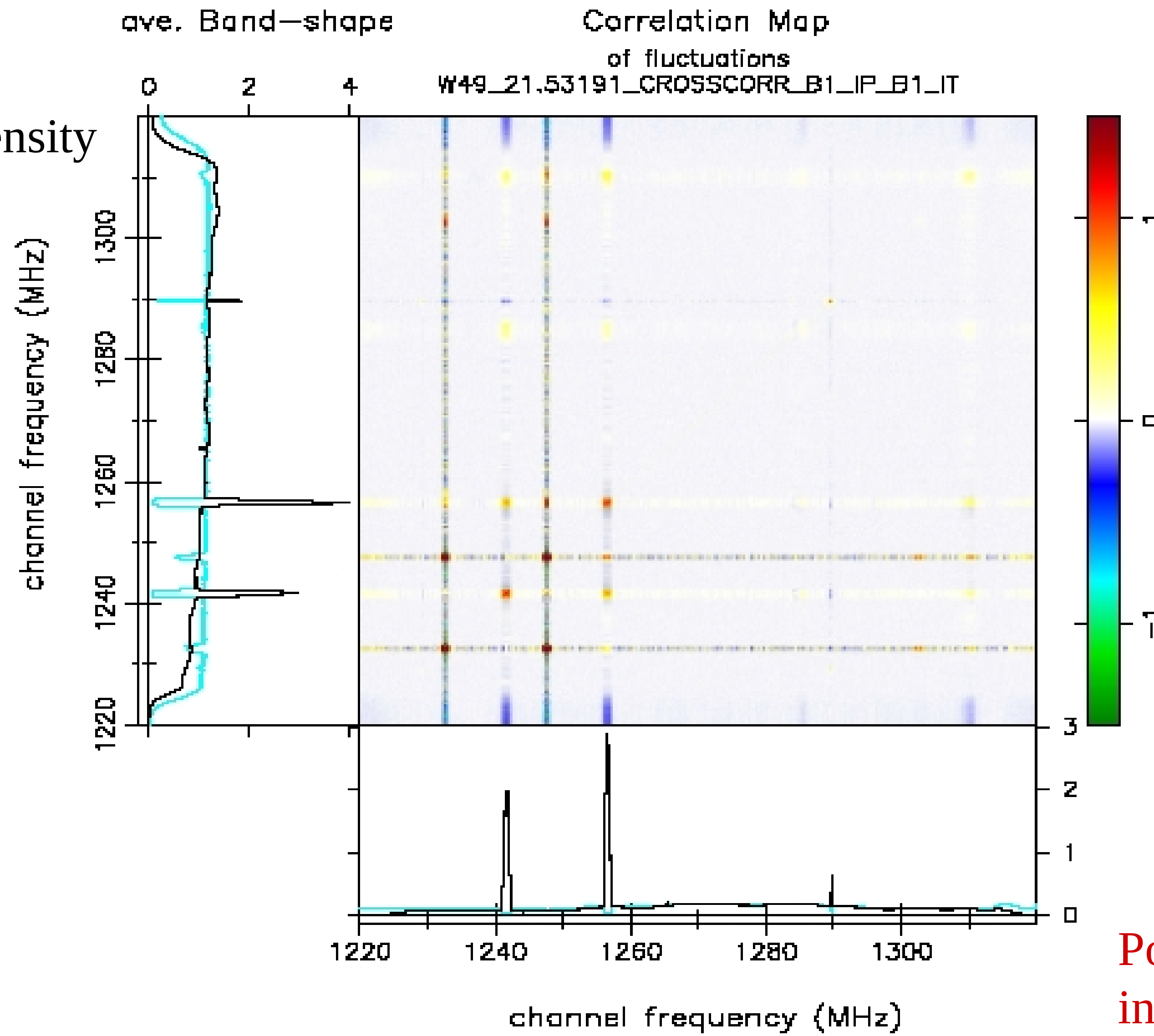
- Side-lobe responses from other (brighter) sources
- Distortions in wave-fronts: local reflections, blockage, aberrations, phase /amplitude corrugations on different scales (e.g. ionosphere)
- Mutual coupling (as in compact phased arrays, or during shadowing; general cross-talk across even electronics pipelines, or any other local common pickups)
- Aliasing and inter-modulation contamination in spectra …. Deviations from linearity; quantization in either in time or in amplitude/phase
- Stray radiation, ground pickups


Average spectrum
Efficiency: rms_ideal/rms_observed
4
2
0
1220
1240
1260
1280
1300
channel frequency (MHz)

Total intensity

Cross-correlation map:

very similar to that for Stokes-I vs Stokes-I correlations



Polarized intensity

Figure 2. Cross-Correlation map of fluctuations in polarized and total intensities. The corresponding average spectra are shown in the bottom and the left panels, respectively.

Exploiting the polarization tag of RFI, and hence, the un-polarized intensity

IUnpol: $I - \mathrm{sqrt}(U^2 + Q^2 + V^2) = I - P$

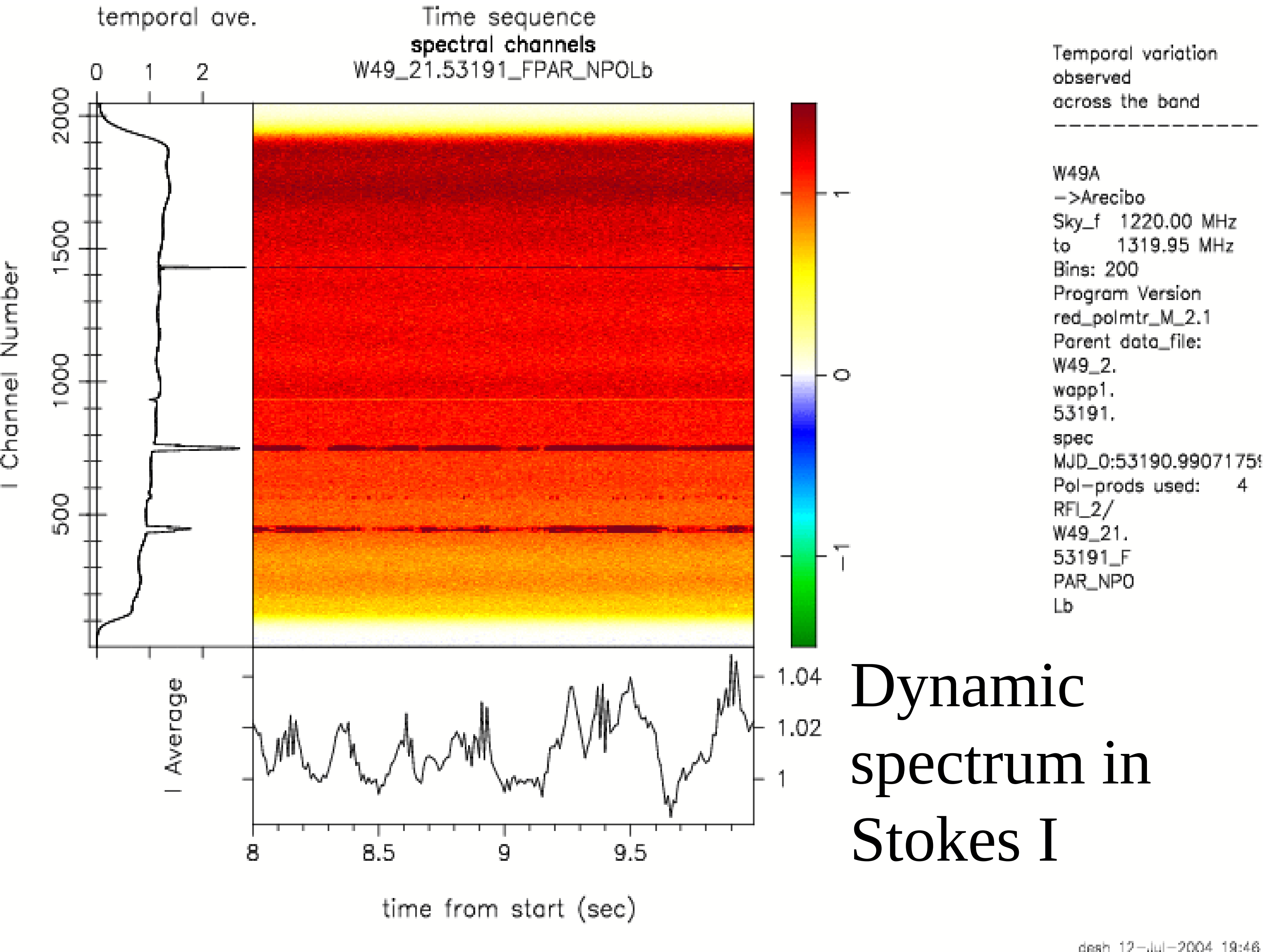
is not to be confused with the invariant quantity

$$I^2 - (U^2 + Q^2 + V^2) = I^2 - P^2 = (I+P)(I-P)$$

which is invariant under relative amplitude and phase calibrations/rotation

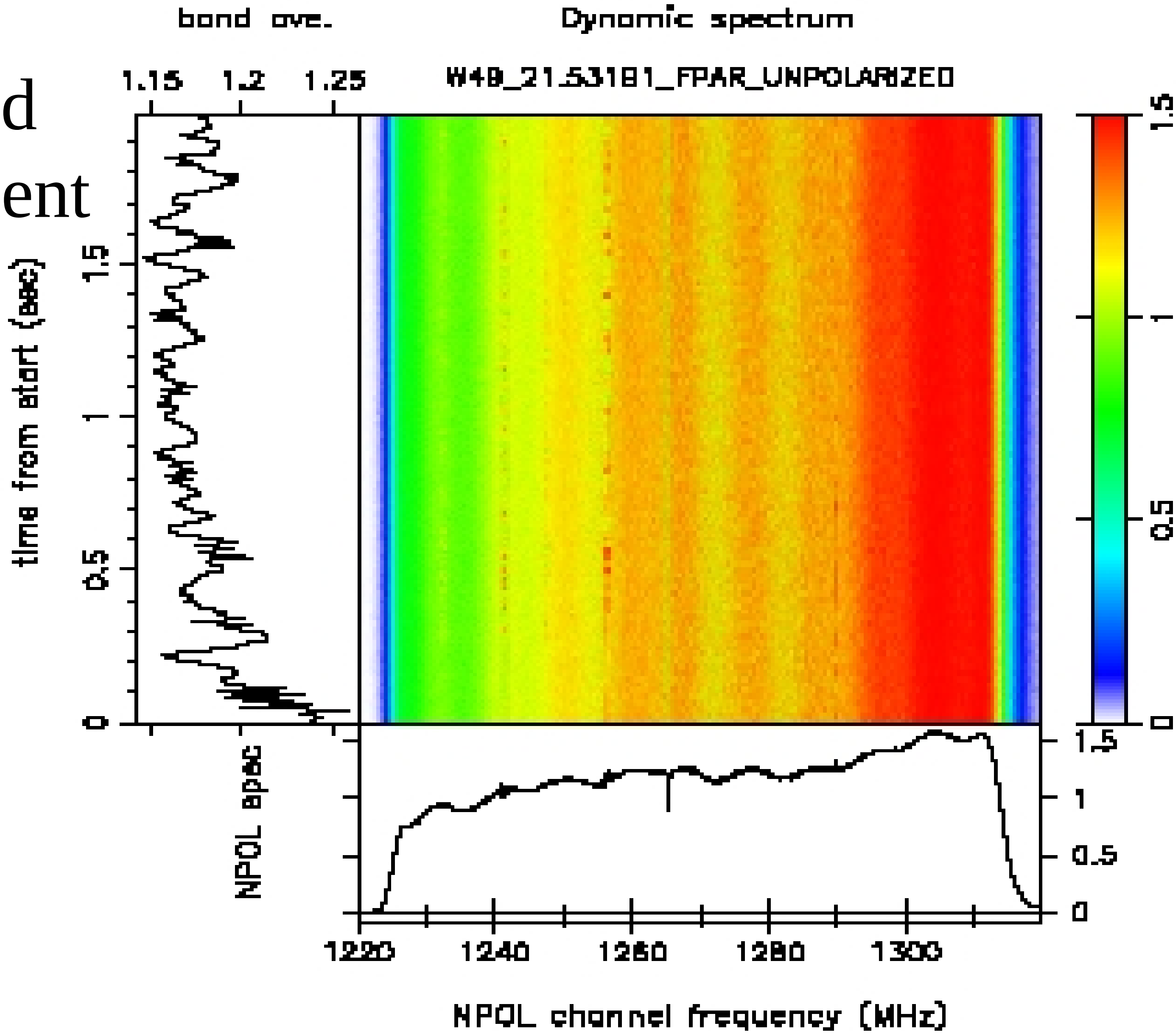
A useful cousin of it is $\dfrac{U^2 + V^2}{I^2 - Q^2}$ or $\dfrac{U^2 + Q^2}{I^2 - V^2}$ (if native) (circular)

or in general $\dfrac{\text{cross-correlation-magnitude-square}}{\text{product of native powers}}$



Dynamic spectrum in Stokes I

Un-polarized component



Propagation of error

Basis for expecting improvement by a factor sqrt(time*bandwidth)

General case of a function f(xi); i=1 to N

distribution functions of xi and <x>

Why oversampling does not help ?

Propagation of errors/uncertainties

Basis for expecting improvement by a factor sqrt(time*bandwidth)

General case of a function f(xi); i=1 to N

distribution functions of xi and <x>

Weighted estimation ? What weightage to use ?

Why oversampling does not help ?

Why even a subtraction of two noisy quantities is as noisy as their addition ?

Systematic vs Random

Systematic distortions, due to non-idealities, (if reversible) can be corrected for, in principle.

If not removed, averaging may or may not help, or sometimes may do even better (i.e. perfect cancellation/removal)

Random noise can be beaten down by averaging, but cannot be “removed” entirely.

Calibration

Undoing the imprint of our measuring device which modifies the signal and adds its own junk. Need to know the modifying imprint (spectral response) as well as one can,

and a measure of the junk added.

Use of reference measurements (but need to watch out for their accuracy as well).

Some damage cannot be repaired.... e.g. spoiling S/N and its initial uniformity.