Self-Calibration in Radio Astronomy

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Insufficiency of Standard Calibration

$$V'_{\text{ij}}(t,u,v) = g_i(t).g_j^*(t).V_{\text{ij}}(u,v)$$

$g(t)$ varies with time during Target source observations.

Main problem from Ionospheric phase variation.
Causes point source to smear out and low level fake structures in a map.

Gain error:
$\sigma_G^2 = \frac{\sigma_V^2}{(N-3)S^2}$ (N is no. of antennas).

Self-calibration

N complex gain errors corrupt $\left(\frac{N(N-1)}{2}\right)$ visibilities.

Allowing N Gains to be variable, provide $\left(\frac{N(N-1)}{2}\right) - N$ constraints.

Self-calibration involves minimising

$$S = \sum_k \sum_{i,j,i \neq j} |V_{ij}(t_k) - g_i(t_k) g_j^*(t_k) . V_{ij}^M(t_k)|^2$$

or

$$S = \sum_k \sum_{i,j,i \neq j} |V_{ij}^M|^2 |X_{ij}(t_k) - g_i(t_k) g_j^*(t_k)|^2 \longrightarrow (1)$$

where $X_{ij}(t_k) = \frac{V_{ij}(t_k)}{V_{ij}^M(t_k)}$.

Steps:

1. Make an initial model of the source using preliminary Cal. and acceptable source structures.

2. Solve for complex gains using Eq.(1).

3. Find the corrected visibility $V_{ij,corr}(t) = \frac{V_{ij}(t)}{g_i(t) g_j^*(t)}$

5. Form a new model from the image made from the corrected data.

6. Go to 2, unless satisfied.

[Averaging time]

How does it work

Consider
$I(l,m) = \frac{1}{M}\sum_{k=1}^{M} V(u_k, v_k) e^{2\pi i(u_k l + v_k m)}$ in 1-D.

$I(l) = \frac{1}{M}\sum_{k=1}^{M} V(u_k) e^{2\pi i(u_k l)}$

Assume $N-1$ antennas to have 0 phases.
1 to have **1 Rad** as phase.

Real source of unit strength at $l=0$.
Recall

$V_{\mathrm{ij}}^{'}(t,u,v) = g_i(t).g_j^{*}(t).V_{\mathrm{ij}}(u,v)$

$I(l) = \frac{1}{M}\Big(\sum_{k=1}^{M-(N-1)} e^{2\pi i(u_k l)} + \sum_{M-N}^{M} e^{i}.e^{2\pi i(u_k l)} \Big)$

→ adaptive optics

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Closure phase & amp

$\Phi_{ijk} = \Phi_{ij} + \Phi_{jk} + \Phi_{ki} \approx 0 + \epsilon$ → noise.

$V: \quad \frac{|V_{ij}(t)|\ |V_{kl}(t)|}{|V_{ik}(t)|\ |V_{jl}(t)|} \sim 1 \pm \epsilon$ → noise