Imaging in Radio Astronomy

by Subhashis Roy

FT reln., DFT & FFT

Sampling fn and dirty Beam

Weighting fn for control of Beam shape

Tapering, ~~Unifo~~ Natural vs Uniform wt.

Gridding the visibility data.

↓

Visibility values assigned to grid points.

Convolution gives predictable result for many random data.

↳ Smoothing with interpolation

$$V_R = R(C * V^W)$$

C ↓ convolution fn.

$$R(u,v) = Ш(u/\Delta u, v/\Delta v) = \sum_{j=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} \delta\left(j - \frac{u}{\Delta u}, k - \frac{v}{\Delta v}\right)$$

$$I_D = F(R) * [(F(C))(F(W^W))] \quad \Big| \quad \Delta u = \frac{1}{FOV(l)}, \Delta v = \frac{1}{FOV(m)}$$

Fourier transform and imaging

$$V(u,v,w) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(l,m)\, I(l,m)\, e^{-2\pi i[ul+vm+w(\sqrt{1-l^2-m^2}-1)]} * \frac{dl\,dm}{\sqrt{1-l^2-m^2}}$$

$$A(l,m).I(l,m) = \int_{-\alpha}^{\alpha}\int_{-\alpha}^{\alpha} V(u,v)e^{2\pi i(\mathrm{ul}+\mathrm{vm})}\mathrm{dudv} \quad \text{—(1)}$$

Holds good if $\Delta\nu.\Delta\tau_g \ll 1$ and $w.(n-1) \ll 1$ or $\underline{w.(l^2+m^2) \ll 1}$.

(1) holds if $V(u,v)$ is a continuous fn.
In practice, it is discrete and uneven.

Rewrite (1) as a DFT relation:

$$I(l,m) = \frac{1}{M}\sum_{k=1}^{M} V(u_k,v_k)e^{2\pi i(u_k l+v_k m)}$$

Requires αN^4 computation

FFT is faster. ($N^2 \log_2 N$)

Map resolution and pixel size

Highest value of u, v.
F.T. relation.
Resolution $\sim (u^2+v^2)^{-0.5}$.

$$I_D = F(S) * F(V')$$
$$I_D = B * F(V')$$

consider point source
$F[V'(l,m)] = \delta(l-l_0, m-m_0)$

point sorc. resp.
$B = F(S) * \delta$
$= F(S)$

F.T. of a 1-D Box is Sinc $\left(\frac{\sin(x)}{x}\right)$.

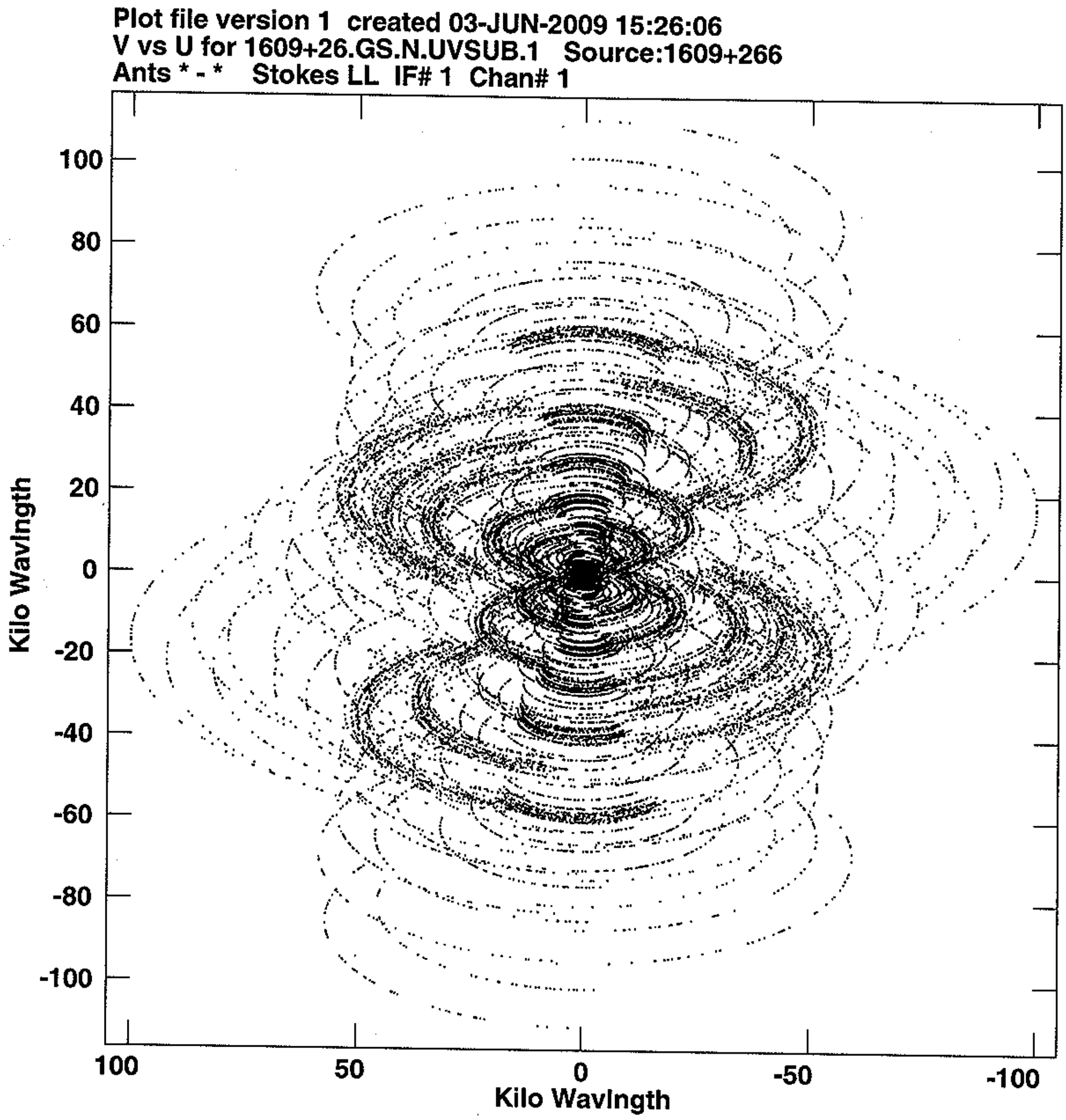

Figure 1. u,v coverage for the source 1609+266 with nearly full synthesis.

How small is $\Delta l, \Delta m$?

Nyquist criterion:

$$\Delta l < \frac{1}{2.u_{\max}}, \quad \Delta m < \frac{1}{2.v_{\max}}$$

$\Delta l, \Delta m$ are pixels (or Cells).

How many pixels in a map ?

Want to cover maximum possible sky area.

Limited by Primary beam size.

$$N_l = \frac{\lambda}{D.\Delta l}.$$

In practice, twice of that for high dynamic range.

F.T. and diffraction pattern of a source

Idea of Beam:

$$S(u,v) = \sum_{K=1}^{M} \delta(u - u_k, v - v_k)$$ [2 D Dirac delta fn]

$$I(l,m) = \frac{1}{M}\sum_{k=1}^{M} S.V'(u_k, v_k)e^{2\pi i(u_k l + v_k m)}$$

$$I_D = F(V^S) = F(S.V')$$

Various assumtions and their effect on the map

Non-coplanar baselines and the 'w' term

$w(l^2+m^2)\not\ll 1$.

Use multiple facets each of which corrects for w term at the facet centre (polyhedron imaging).

Frequency channel averaging and Bandwidth smearing

u,v varies due to finite bandwidth.

Time averaging of data and source smearing

Source $V(u,v)$ changes with 't'.

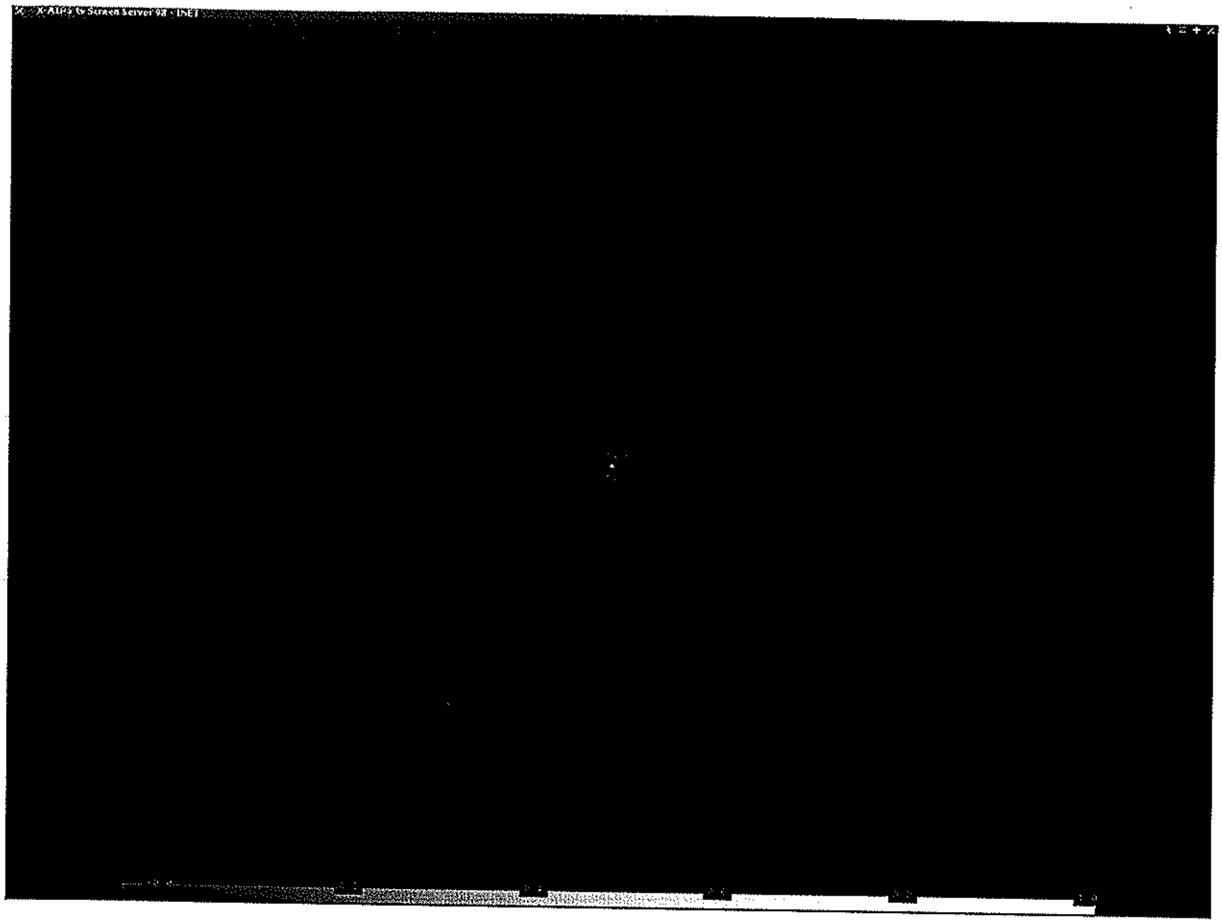

Figure 2. Beam pattern for the full synthesis data of GMRT on 1609+266.

Various data Weights

Data weights changes Beam pattern to reduce Diffraction Sidelobes.

Data Tapering (T_k), Density weight (D_k) and reliability weight (R_k):

Tapering: Reduced contribution from edges.

Density weight:
Data from different parts of u,v plane gets uniform weight (uniform weighting).
Density weight=1 (Natural weighting).
Reliability weight:
More noisy data from a few antennas get reduced weight.

$S^W(u,v) = \sum T_k.D_k.R_k.\delta(u-u_k, v-v_k)$.

Taper density reliability

$T_K = T(r_K)$, where $r_K = \sqrt{u_K^2 + v_K^2}$

typically Gaussian.

$T_r = \exp\left(-\frac{r^2}{2\sigma^2}\right)$

$D_K = 1 \rightarrow$ Natural weighting

$D_K = \frac{1}{N_s(K)} \rightarrow$ uniform "