

Clarifying Confusion

SPARCS 2016 Goa

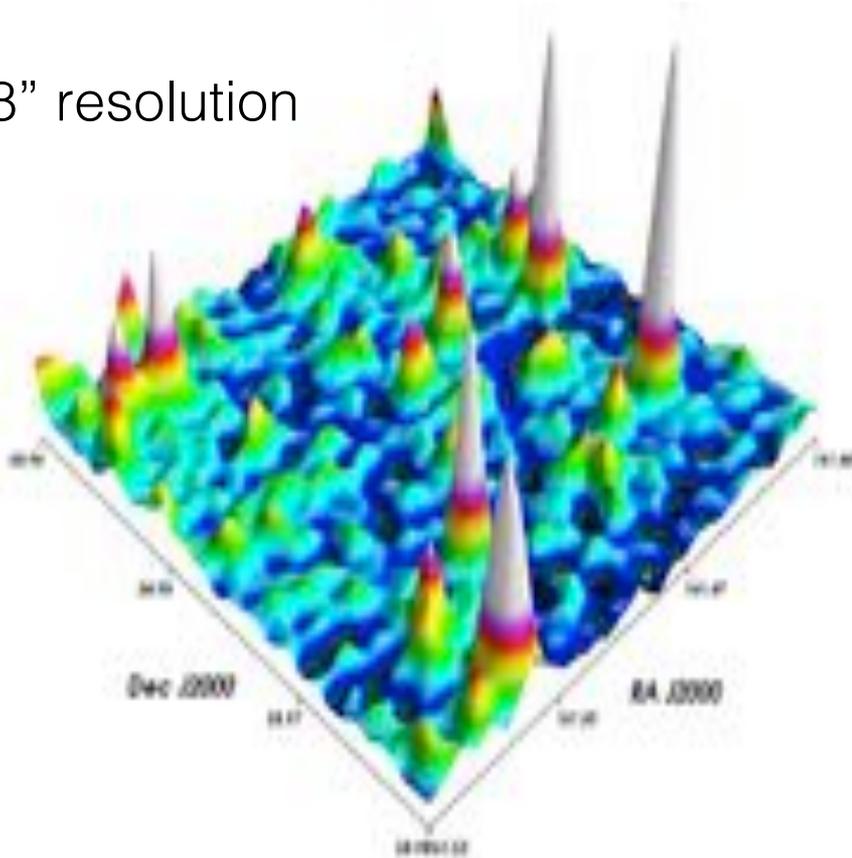
Tessa Vernstrom

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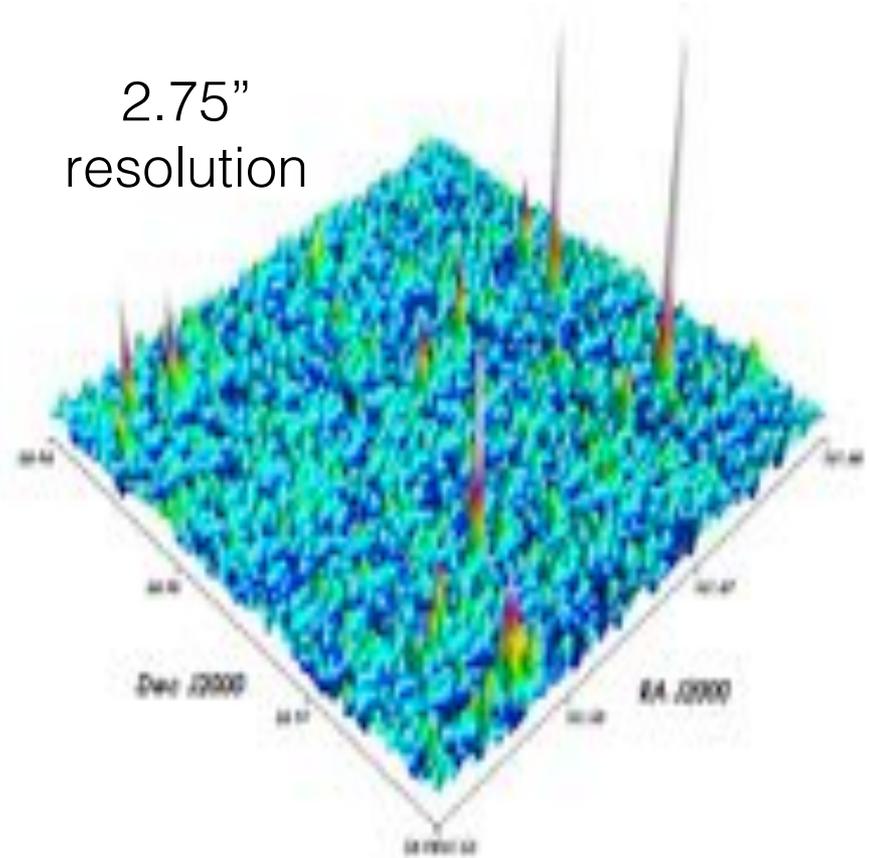
What is Confusion?

- Confusion is the blending of faint sources within a telescope beam

8" resolution

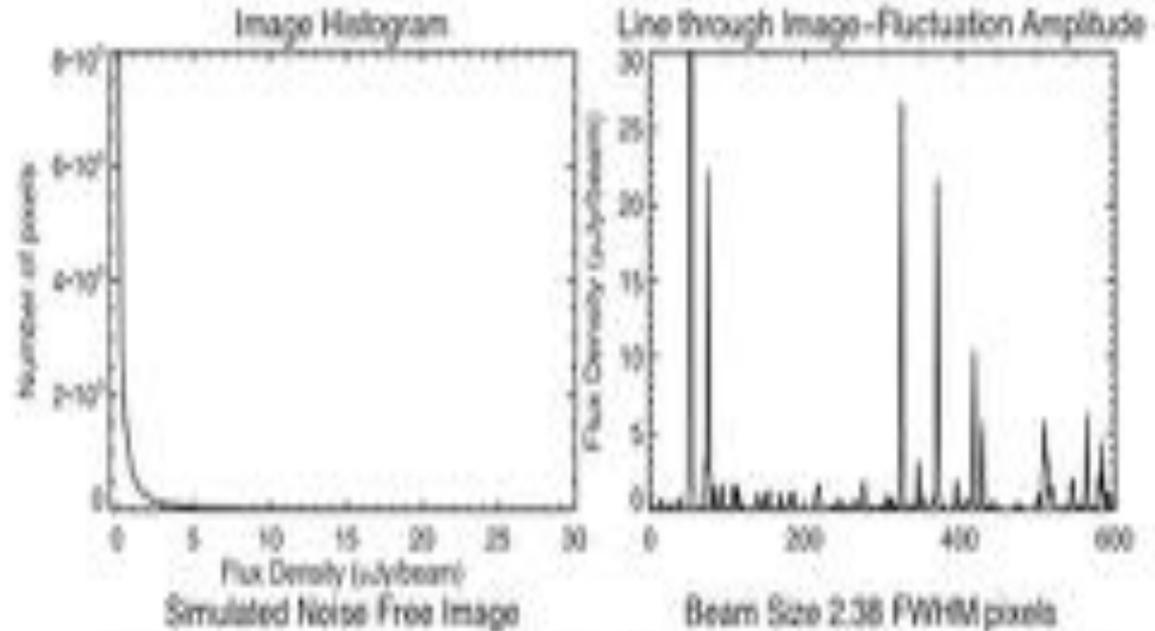


2.75" resolution



What is Confusion?

- Confusion is the blending of faint sources within a telescope beam
- PDF of image pixel histogram from confusion known as $P(D)$
- Confusion noise, σ_c (width of $P(D)$)
 - governed by beam and source count



Why is confusion important?

- Discrete source count
- Diffuse emission detection
- The cosmic radio background (CRB)
- Catalogue fit uncertainties
- Angular power spectrum / Fourier domain
- Luminosity functions / galaxy evolution and frequency dependence
- Foreground characterization

How? Probability of Deflection

- Fitting of Image histogram \rightarrow statistical estimate of source counts as faint as $\sim \sigma_c$

- Input

- Source count model
- Pixel size, beam shape
- Instrumental noise

- Mean density of observed flux

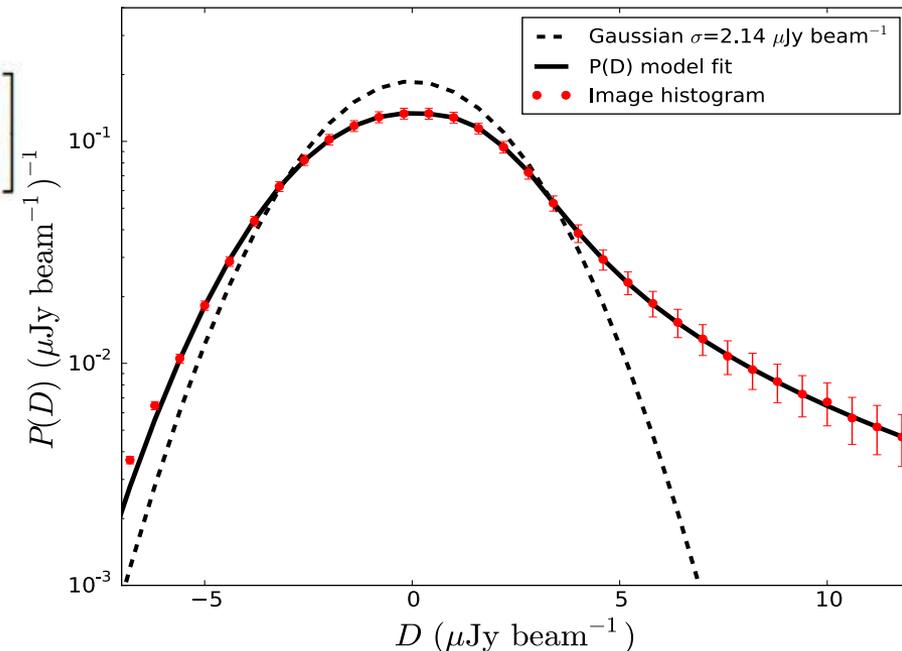
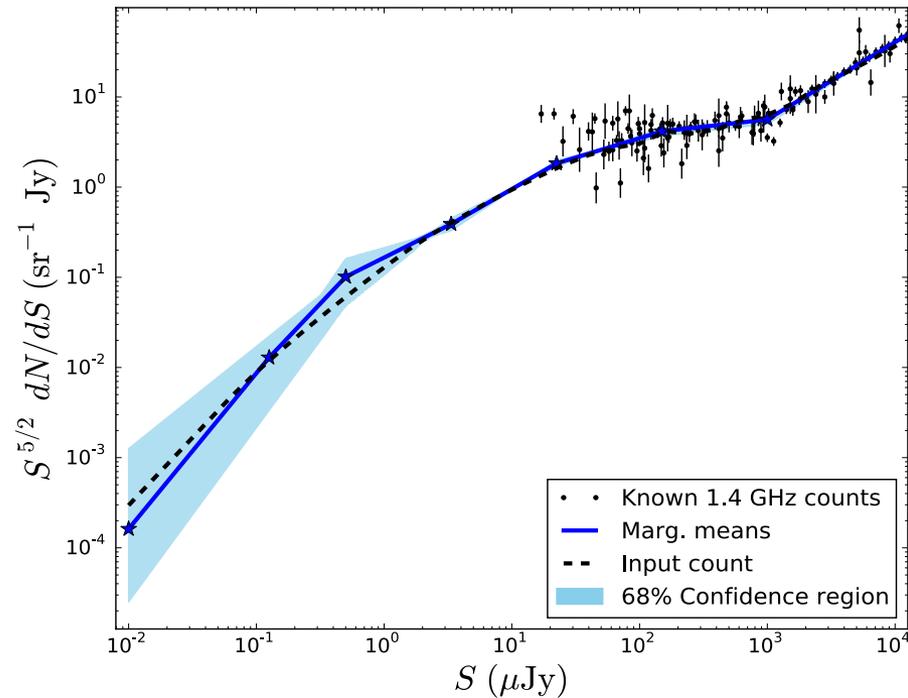
$$R(x) dx = \int_{\Omega} \frac{dN}{dS} \left(\frac{x}{b}\right) b^{-1} d\Omega dx$$

- Can use any continuous source count model

$$P(D) = \mathcal{F}^{-1} \left[\exp \left(\int_0^{\infty} R(x) e^{iwx} dx - \int_0^{\infty} R(x) dx - i\mu w - \frac{\sigma_n^2}{2} w^2 \right) \right]^{-1}$$

- Node model

- Fixed position in Log(S)
- Fit amplitude of node in Log(dN/dS)
- Interpolate between nodes
- Set of connected power-laws



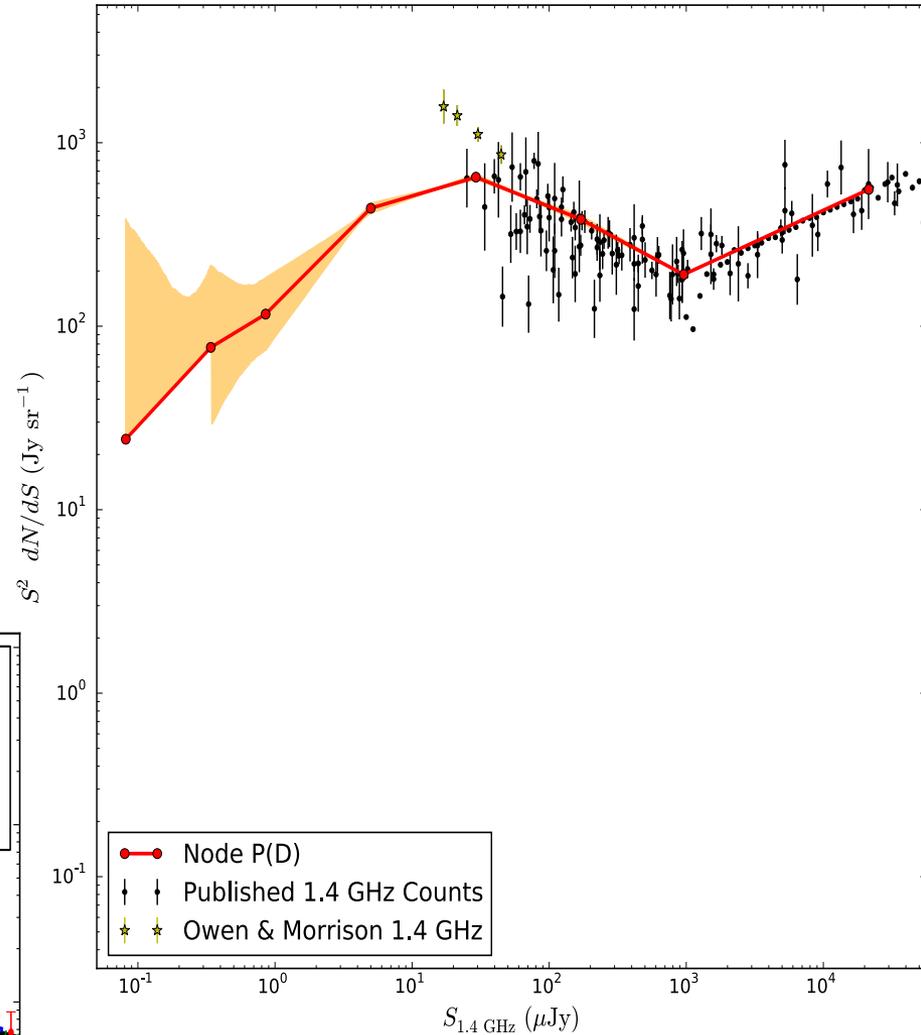
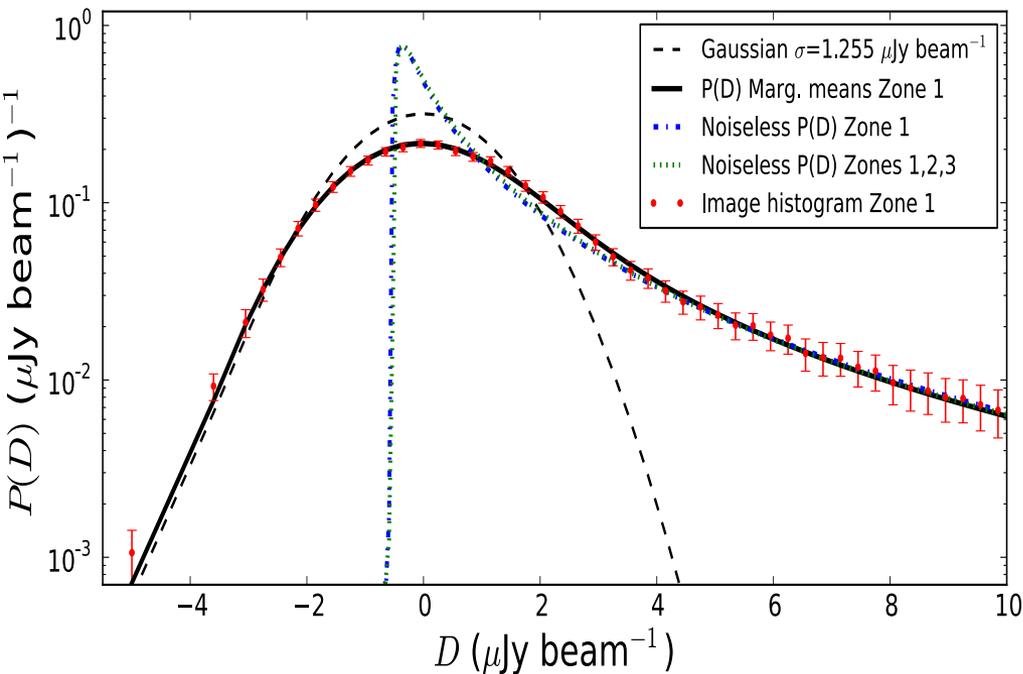
Discrete Source Count

- JVLA
 - Lockman Hole North
 - 3 GHz single pointing
 - Rms $\sim 1 \mu\text{Jy}/\text{beam}$
 - 8 arcsec beam



Discrete Source Count

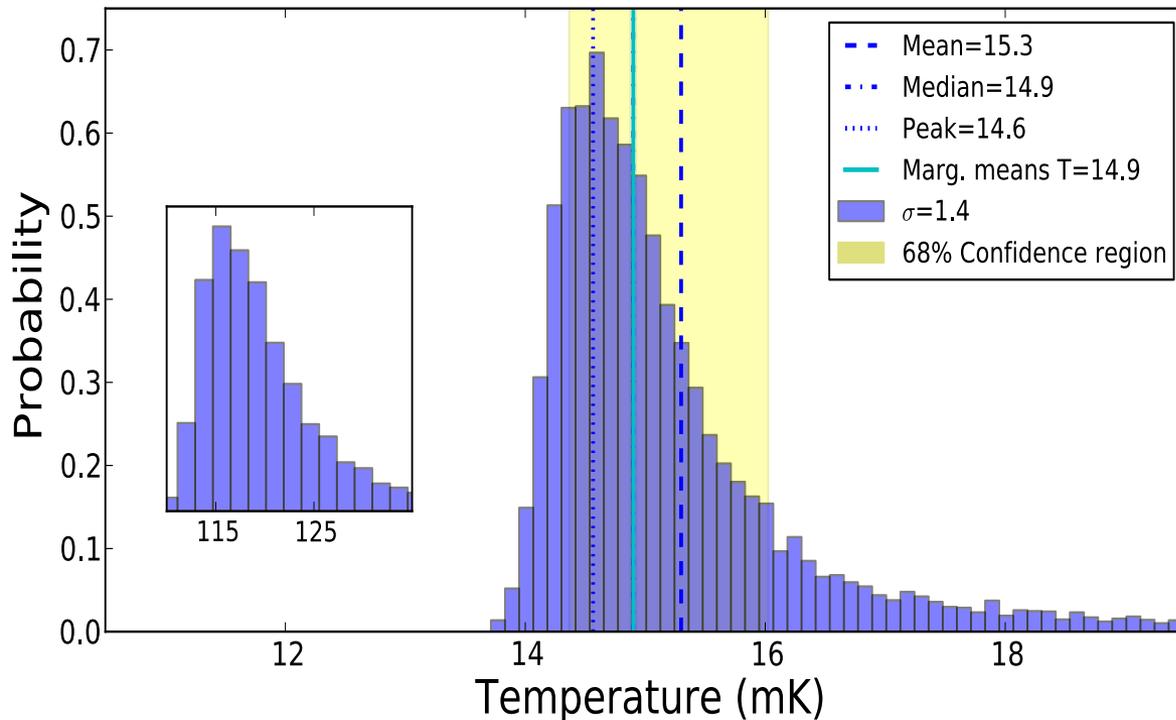
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Cosmic Radio Background

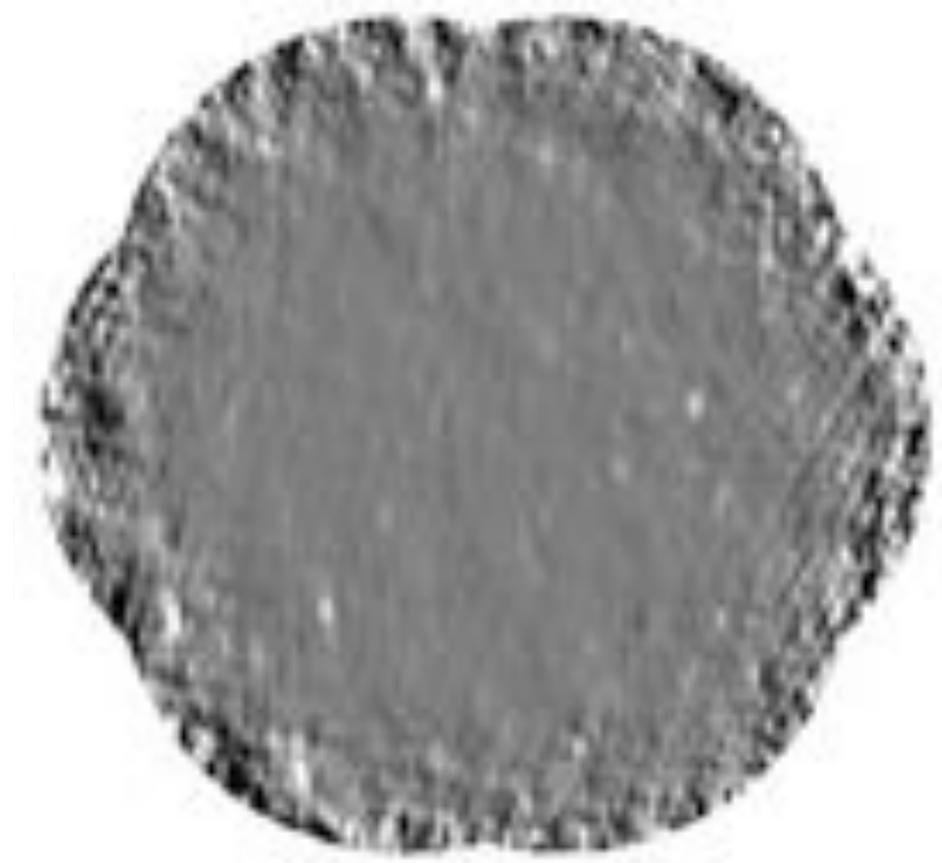
- Background temperature, T_b , from extragalactic sources by integration source count

$$\int_{S_{\min}}^{\infty} S \frac{dN}{dS} dS = \frac{T_b 2k_B \nu^2}{c^2}$$



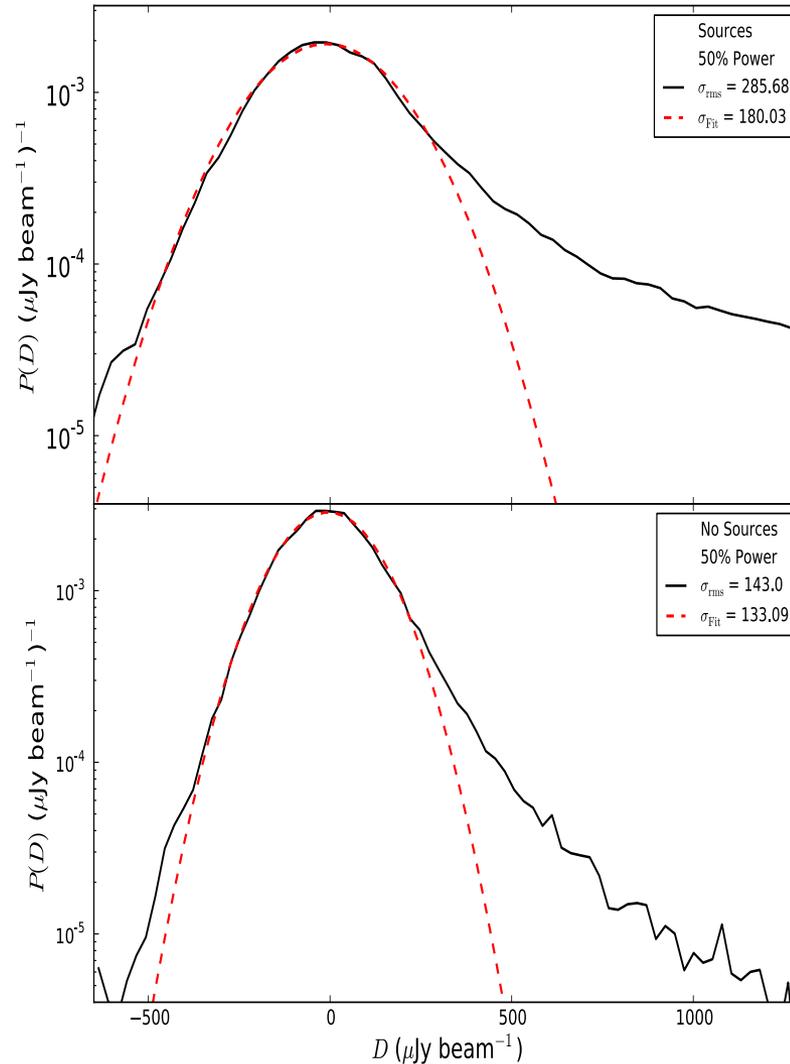
Diffuse Emission

- Can statistically detect presence of diffuse sources : galactic haloes, cluster haloes, relics, cosmic web
- Subtract point sources or use discrete source count model
- Example: ATCA
 - ELAIS S1
 - 7 pointing mosaic
 - 1.7 GHz
 - 150" x 60" beam
 - RMS ~ 50 μ Jy
- Use ATLAS point source models to subtract bright sources and JVLA discrete count for un-subtracted sources



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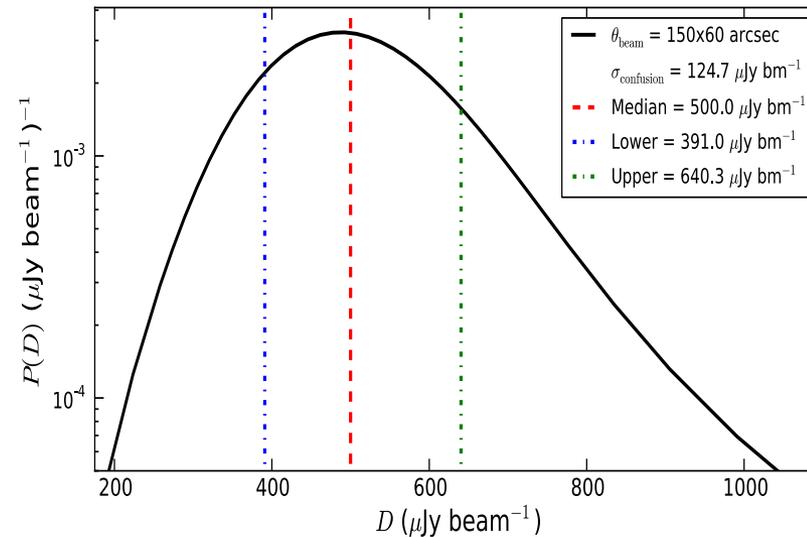


All Sources

Bright sources
subtracted

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Model $P(D)$ of faint sources

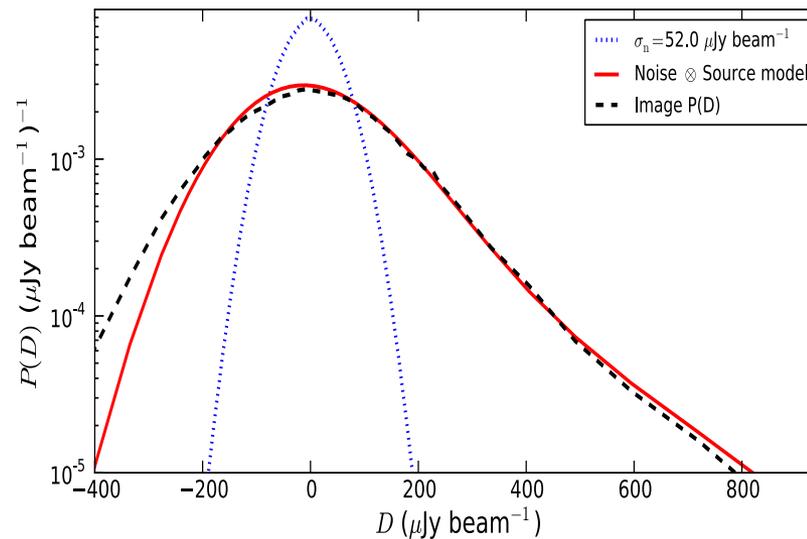
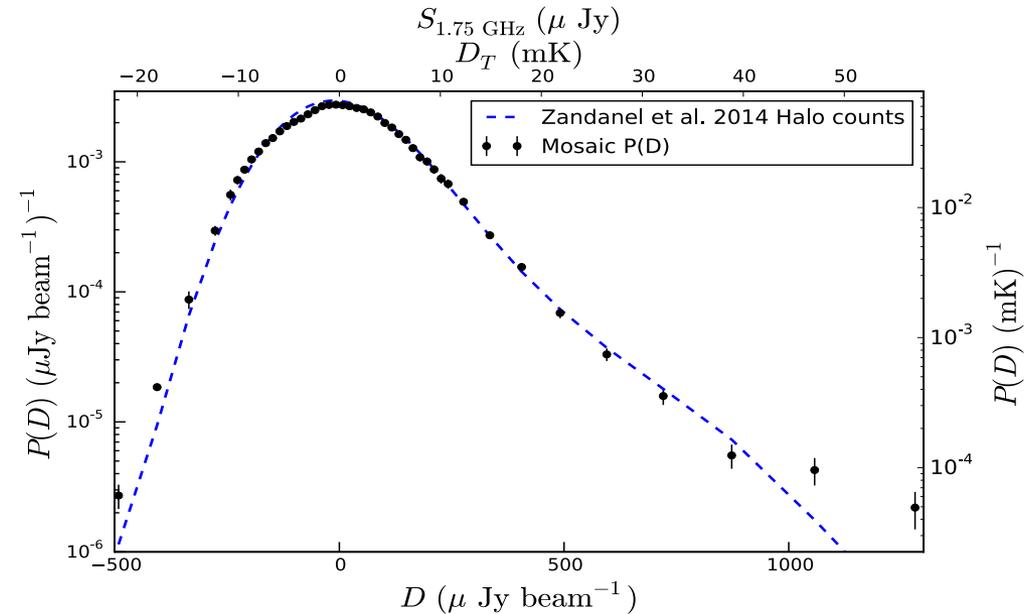
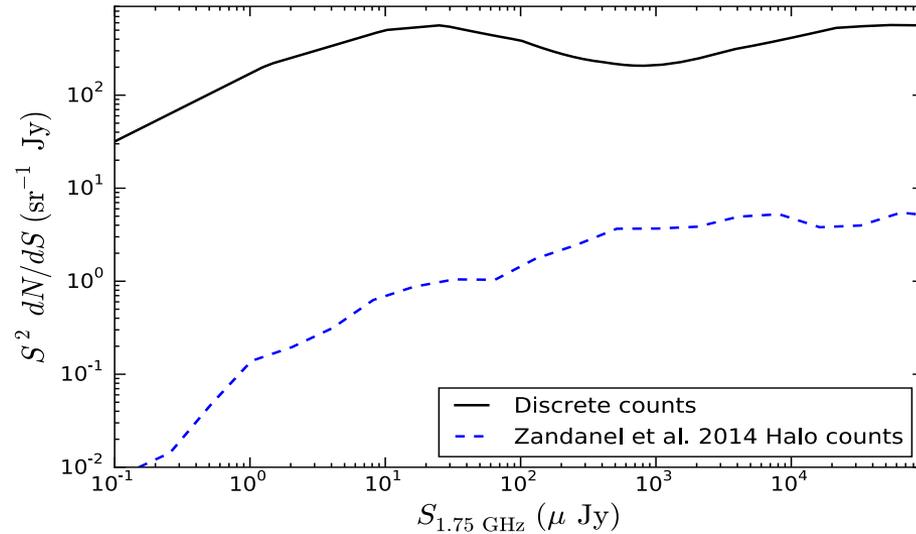


Image $P(D)$ compared to model
 ← 3σ difference

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- Fit models of diffuse source count – dark matter haloes or clusters etc.



Catalogues and Counts

- Find, fit, and count number of source with $S > 5\sigma$

- But σ is total noise σ_{total}

$$\text{where } \sigma_{\text{total}}^2 = \sigma_{\text{instrumental}}^2 + \sigma_{\text{confusion}}^2$$

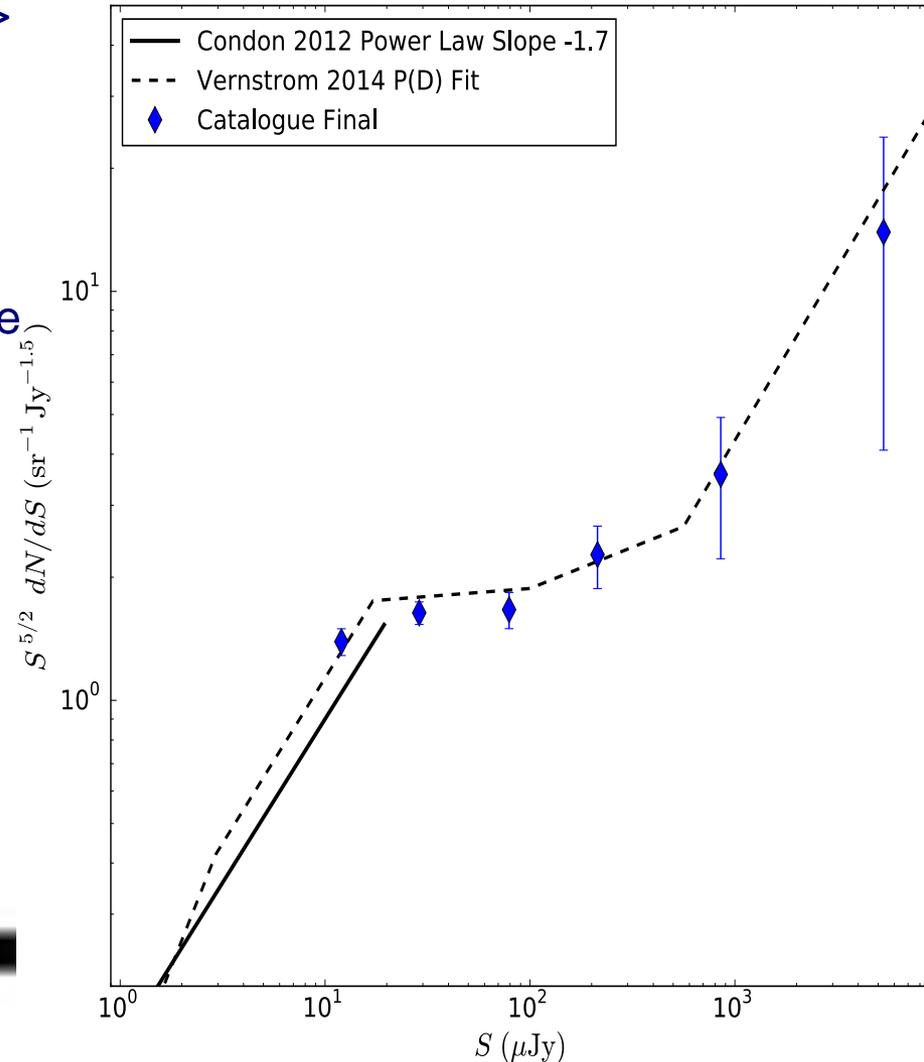
- Need to know confusion noise to determine cutoff

- Also for fit uncertainties

$$p^2 = \frac{\pi}{8 \ln 2} \frac{\theta_M \theta_m S_{\text{peak}}^2}{h^2 \sigma_{\text{total}}^2}$$

- False detection rate

- Can also compare catalogue count and $P(D)$ count



Angular Power Spectrum

- Can use the confusion noise to estimate Poisson contribution

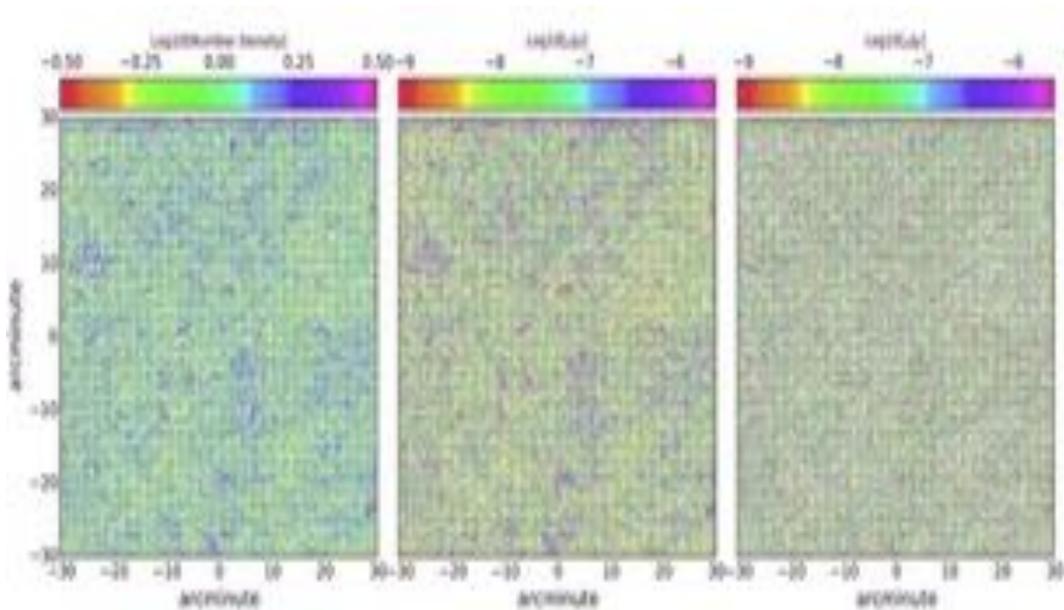
$$\sigma = \int S^2 \frac{dN}{dS} dS$$

- Where σ^2 is the (flat) amplitude of the power from sources
- Also known as P(D) in the visibility plane
- Simulation: SKADS source fluxes at 1.4 GHz
 - Randomly distributed positions and fluxes
 - Positions and fluxes clustered according to a power law

Angular Power Spectrum

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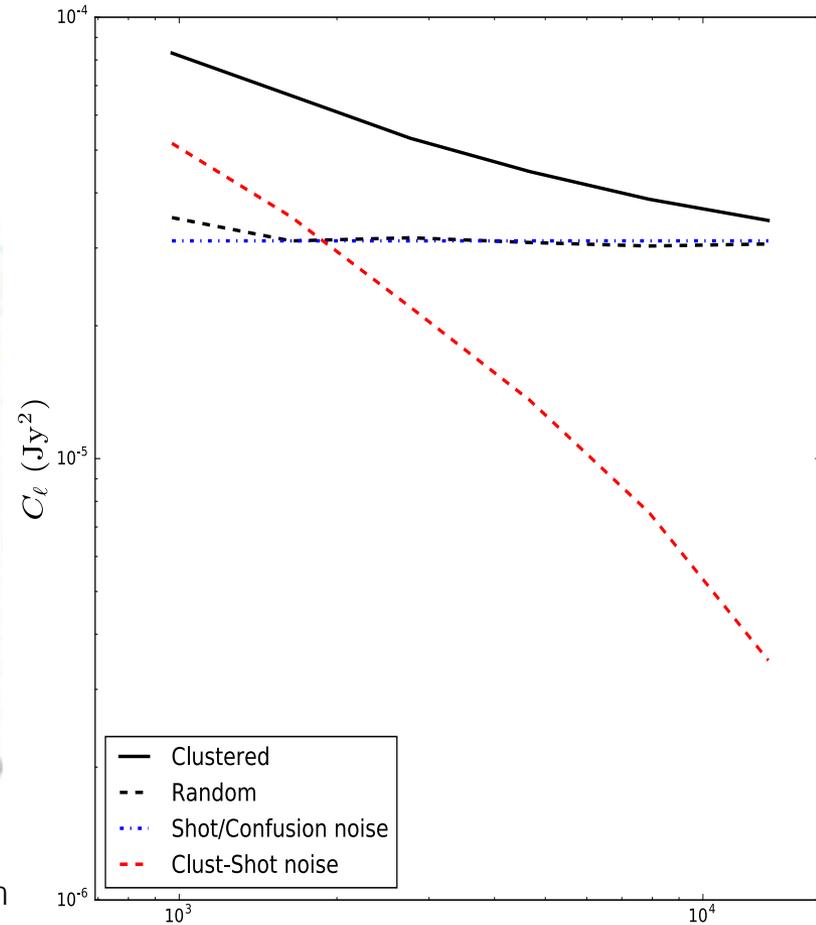
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Number density clustered

Flux density distribution clustered

Flux density distribution random

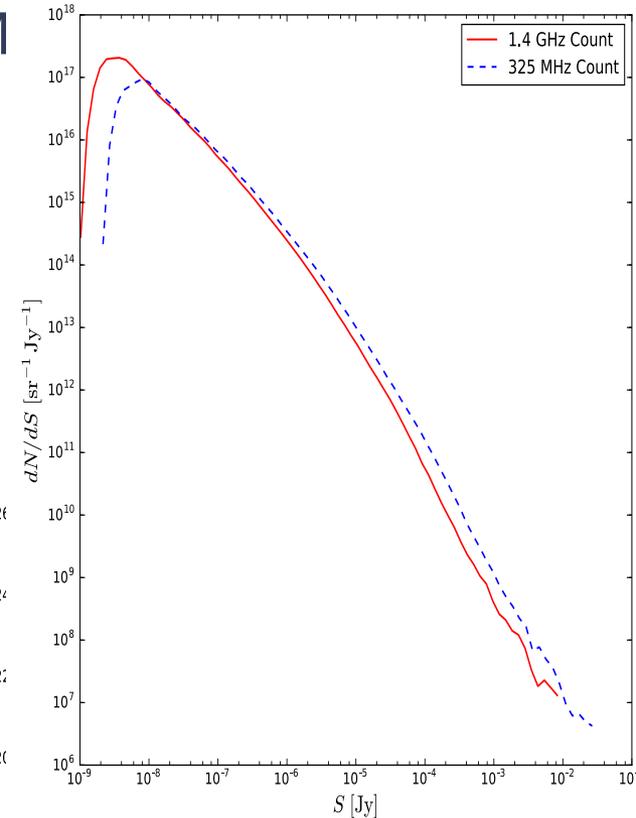
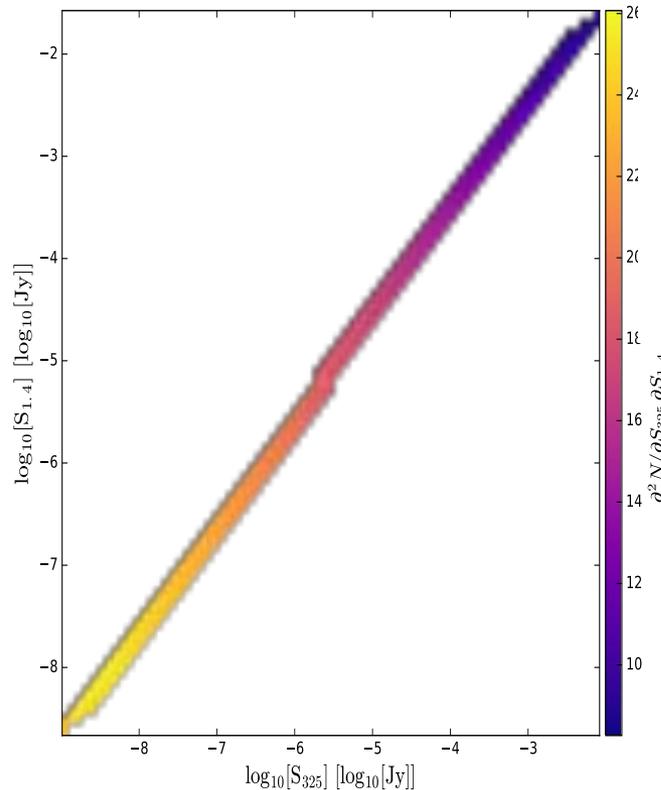


2D P(D) – Luminosity, Evolution, Population M

- Fit 2D histogram
 - Two Frequencies or Total & Polarised Intensity
 - How correlated is the confusion noise between frequencies
 - Uses more information to obtain tighter constraints
- Model: 2D source count at 2 frequencies
 - Can fit multiple population luminosity functions and spectral indices to predict source count at both frequencies
- Simulation using 1.4 GHz – 8 arcsec beam and 325 MHz – 15 arcsec beam (VLA vs GMRT)

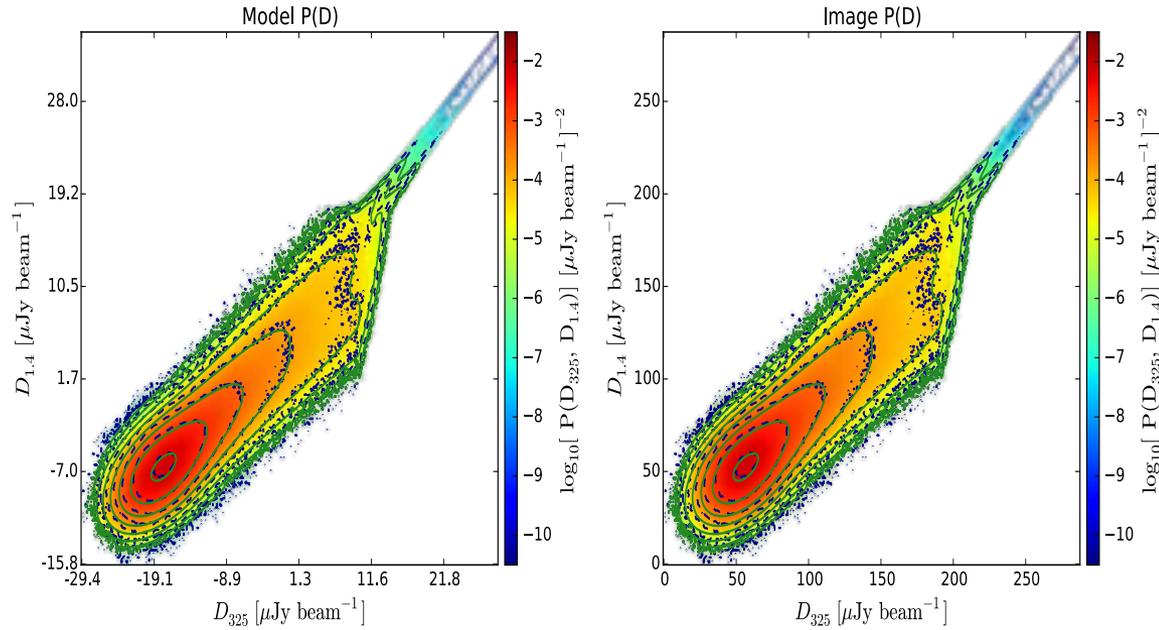
Two populations, AGN
 $\alpha = -0.85$, Star-forming
 $\alpha = -0.6$

$$\log_{10} \left[\frac{\partial^2 N(S_1, S_2)}{\partial S_1 \partial S_2} \right]$$



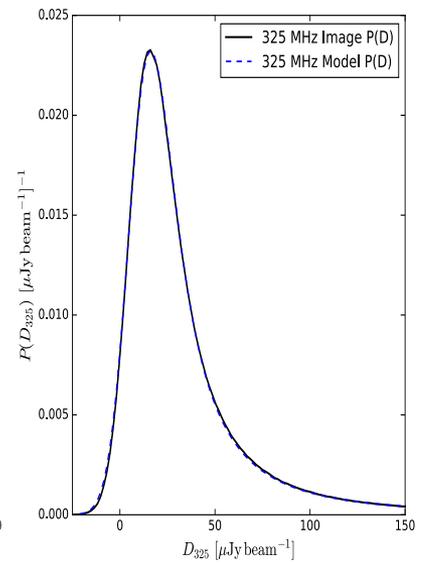
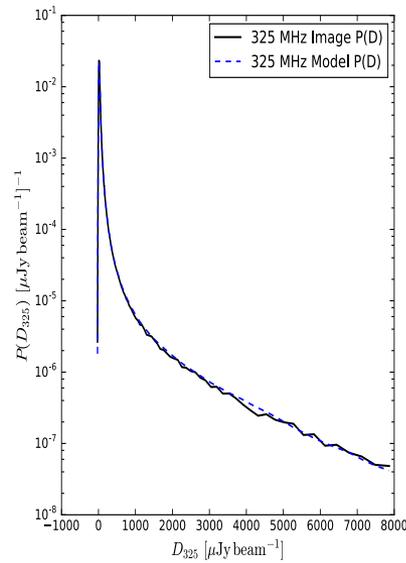
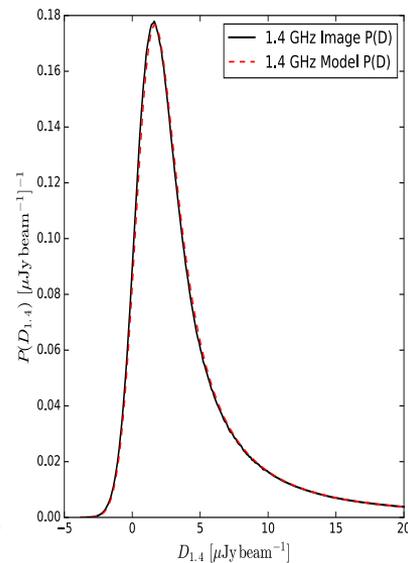
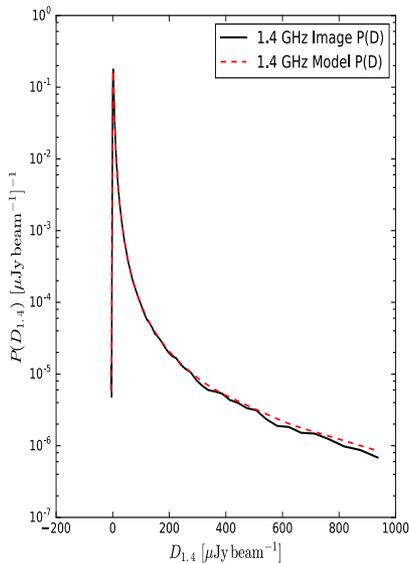
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• Similar

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f_i

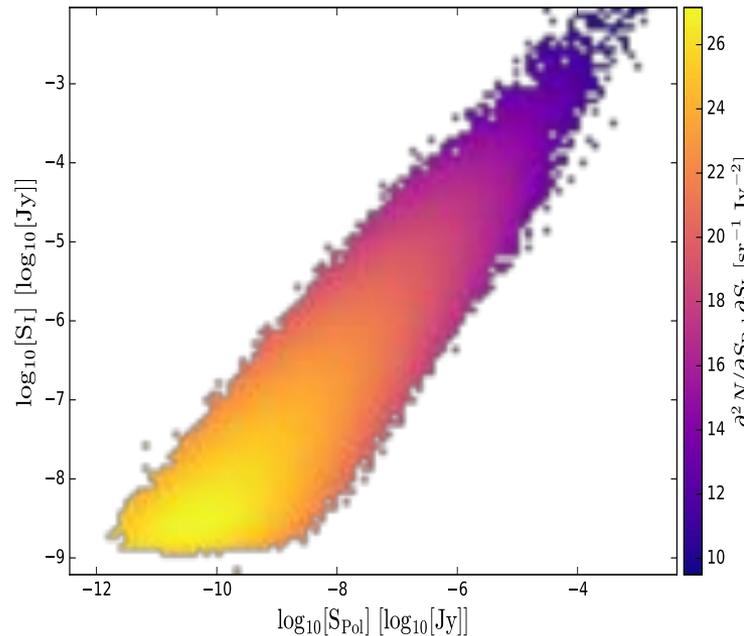
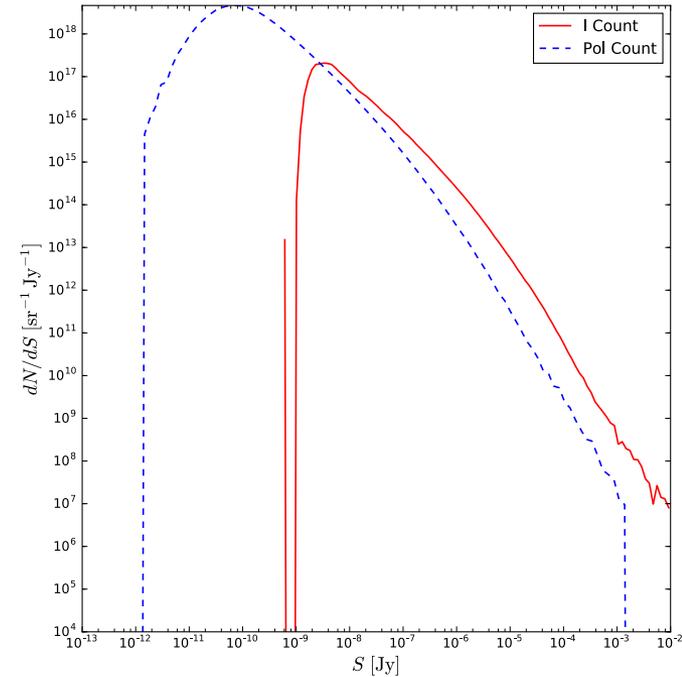


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- Simulation using 1.4 GHz – 8 arcsec beam and Total intensity I and Polarised P

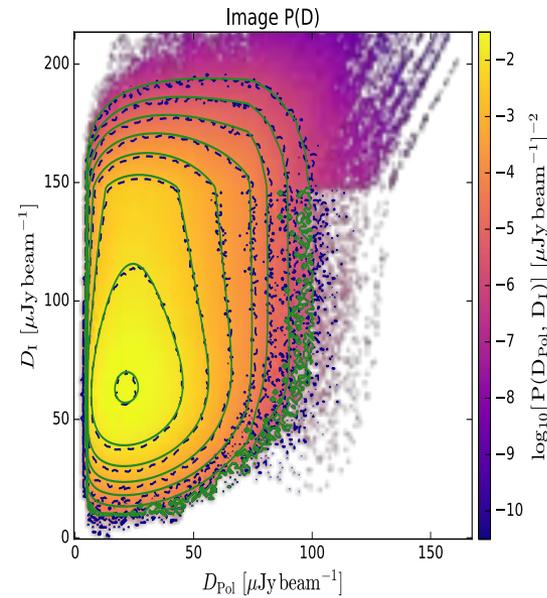
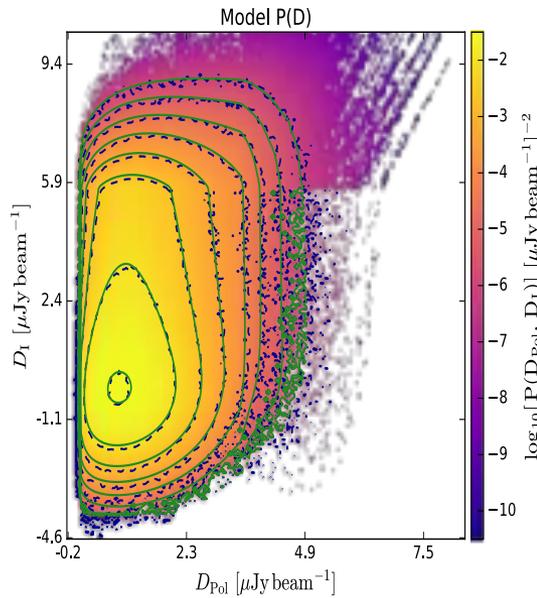
Total Intensity and Polarised Intensity – $dN/dP(S)$
 Polarised count – 2 parameter function of Stokes I count

$$\log_{10} \left[\frac{\partial^2 N(S_1, S_2)}{\partial S_1 \partial S_2} \right]$$



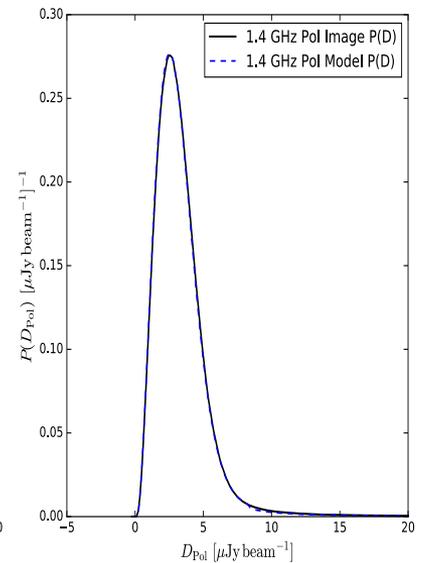
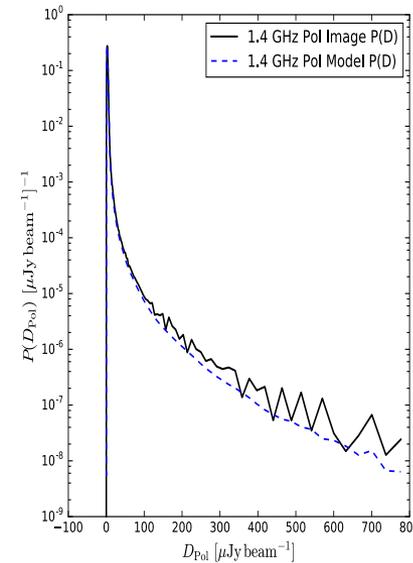
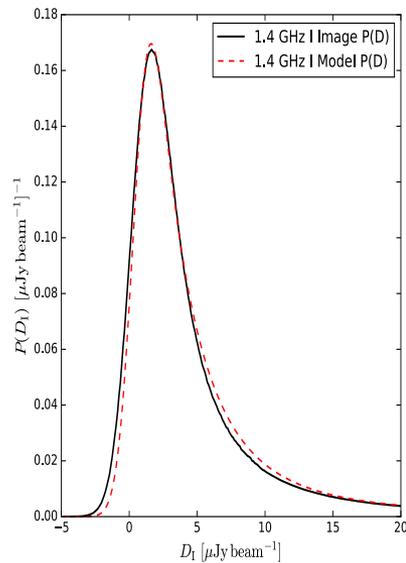
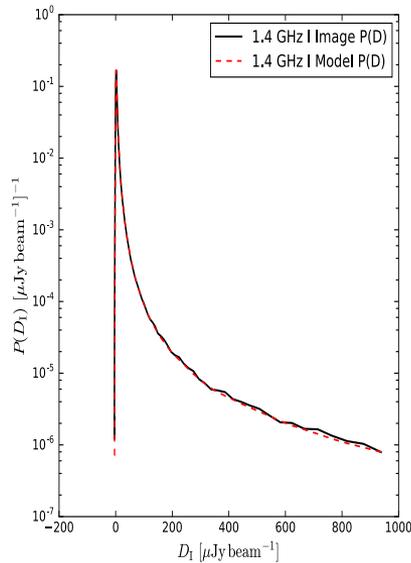
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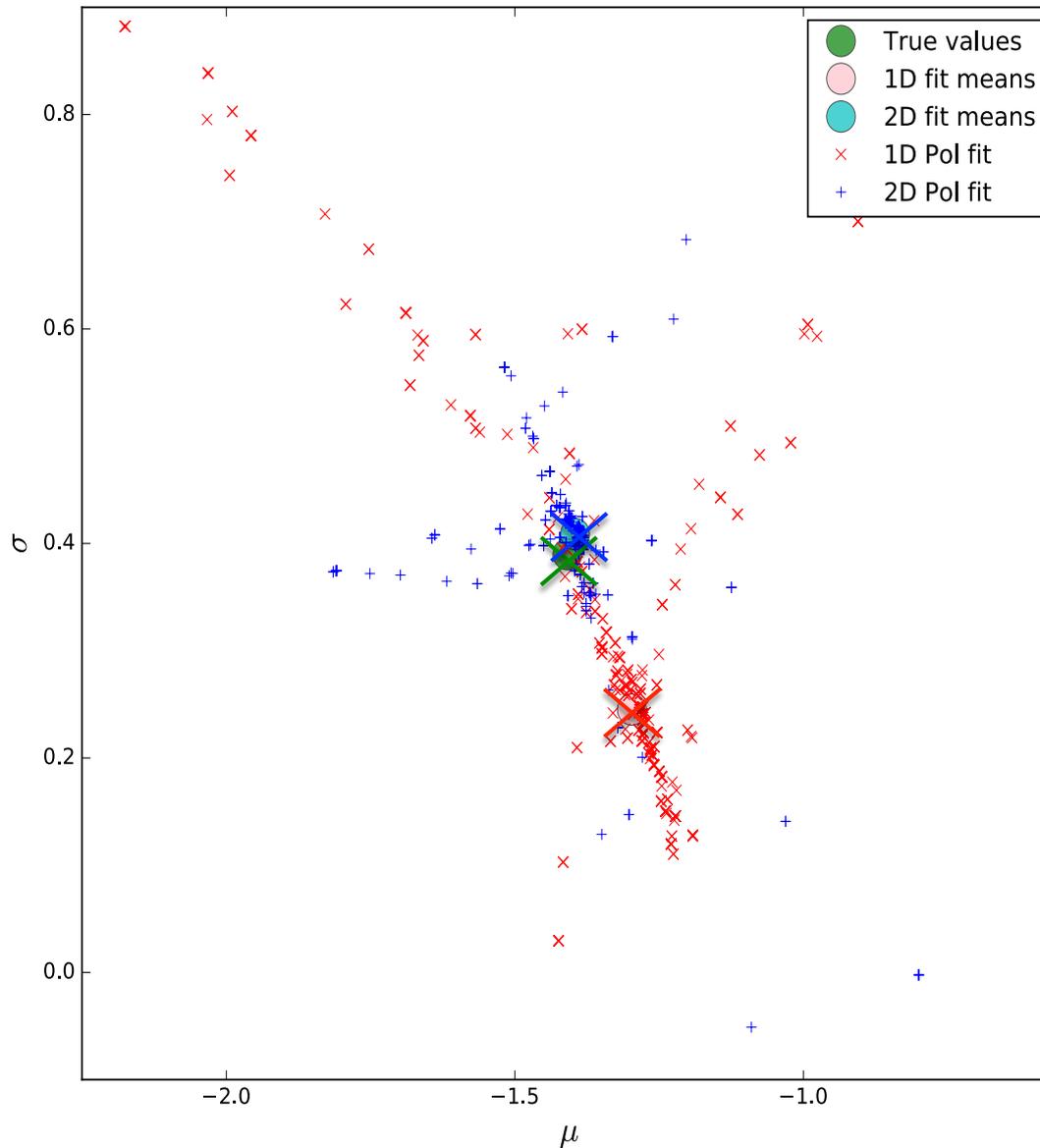
- Sim arc

DUI for

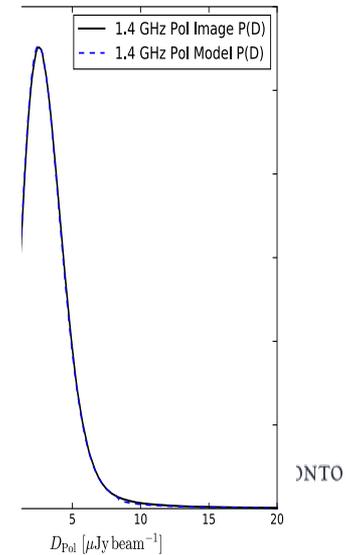
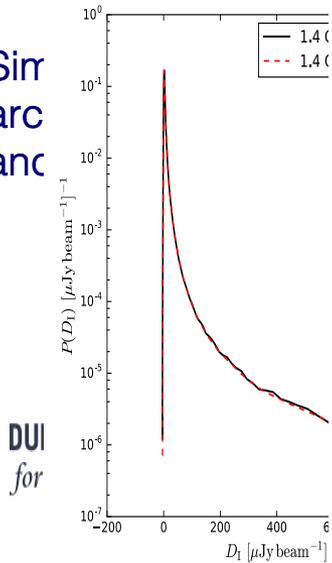


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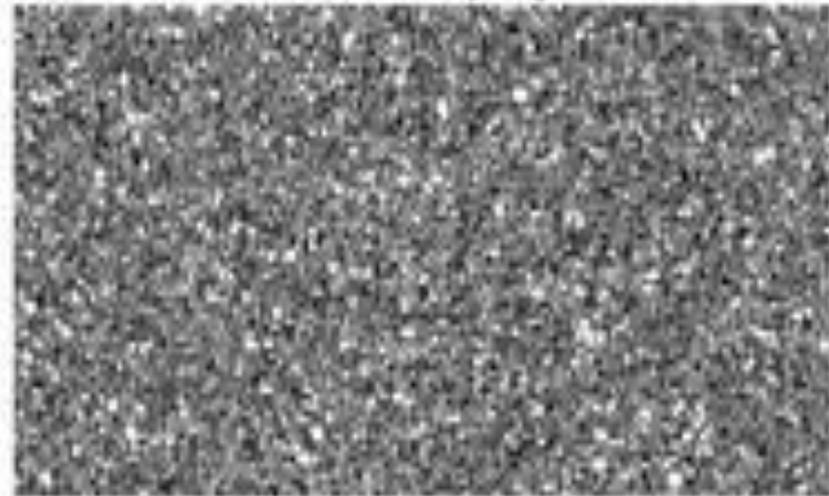
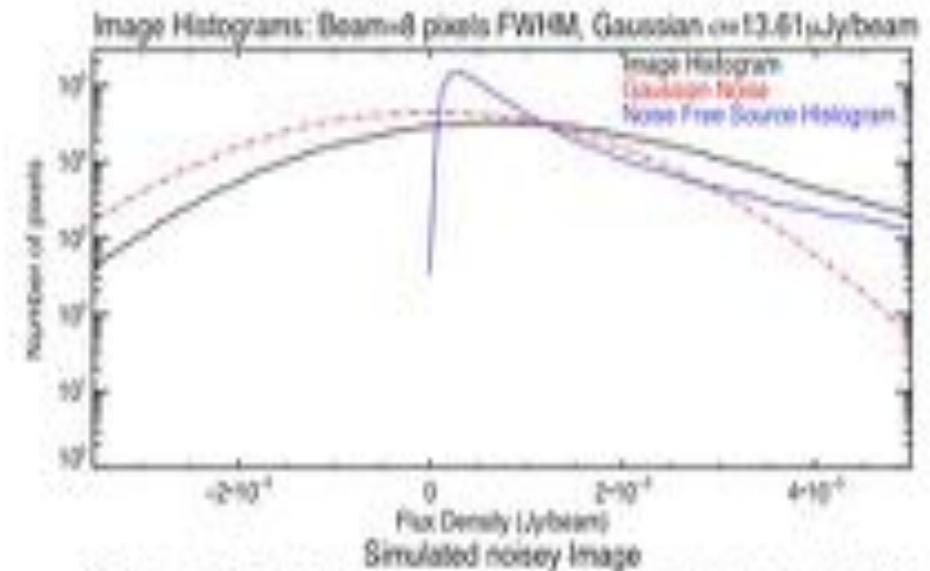


- Simulated data



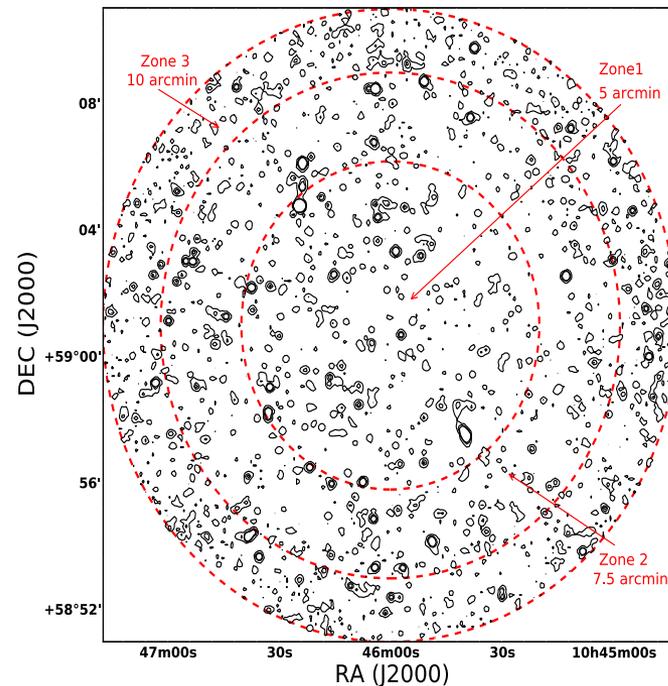
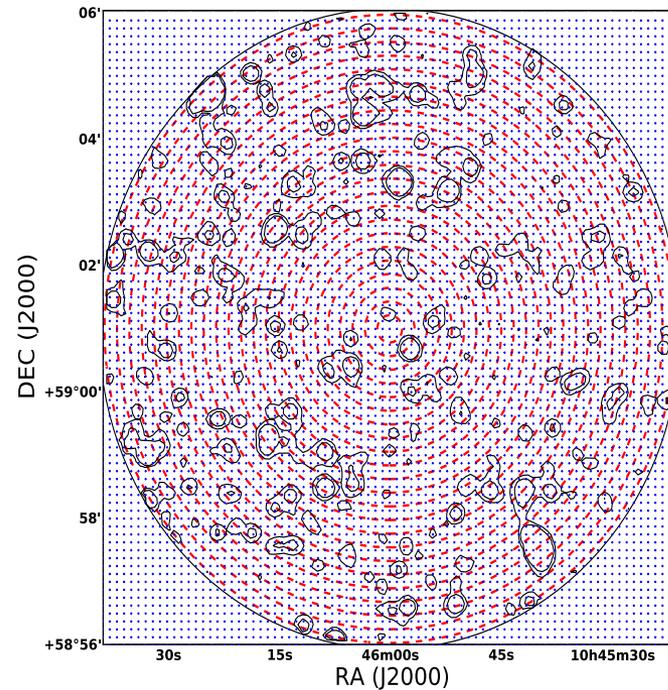
Issues – Instrumental Noise

- $\sigma_t^2 = \sigma_n^2 + \sigma_c^2$
- For faint counts want $\sigma_c > \sigma_n$



Issues – Instrumental Noise

- P(D) assumes constant instrumental noise
- Not the case with primary beam corrections or mosaics
 - Use multiple zones where change in noise is small
 - Within those zones create weighted histogram
 - Weights $\propto \frac{1}{\sigma_i^4}$
- Example: MWA/SKADS simulation
 - 150 GHz $\sim 35^\circ$
 - 2 arcmin Gaussian beam
 - Single 2 minute snapshot
 - $\sigma = 0.02$ Jy



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Just sources

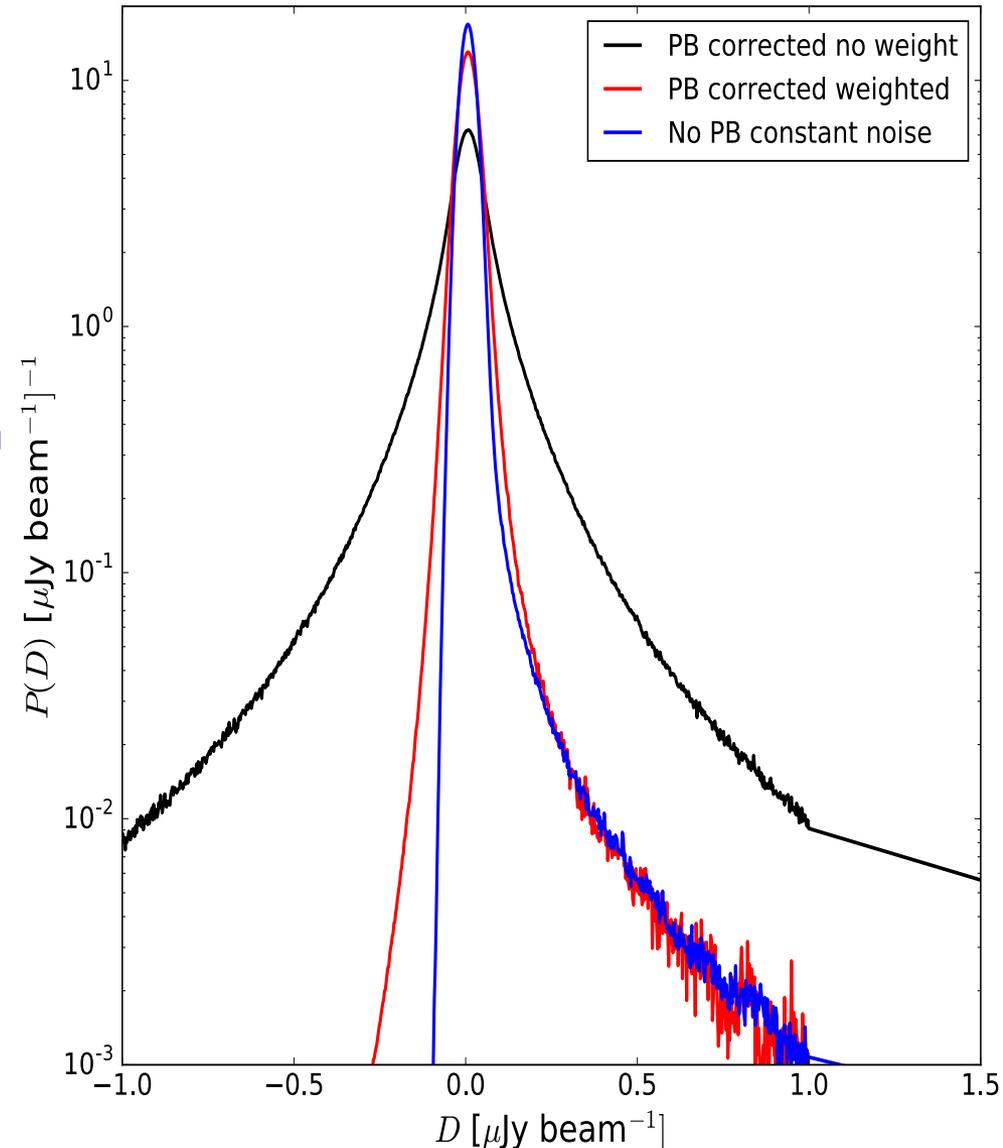
$$\propto \frac{1}{\sigma_i^4}$$

sources*primary beam
+noise

(sources*primary beam
+noise)/primary beam

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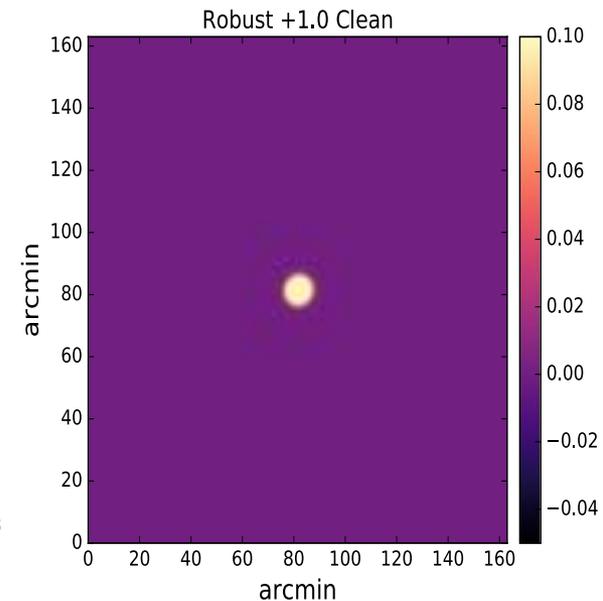
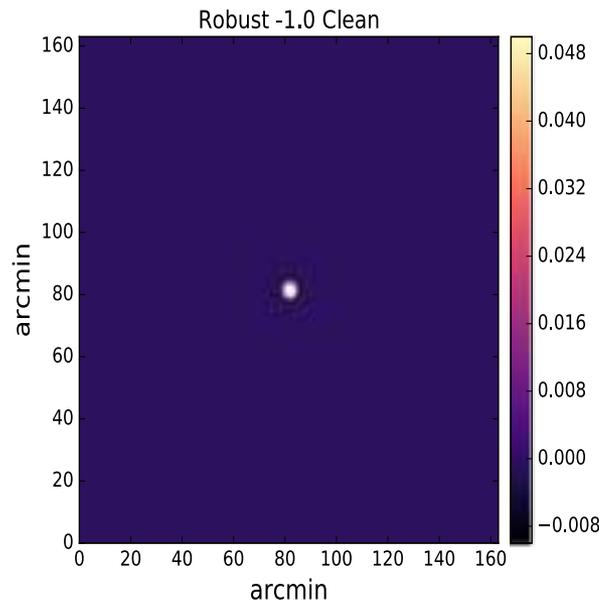
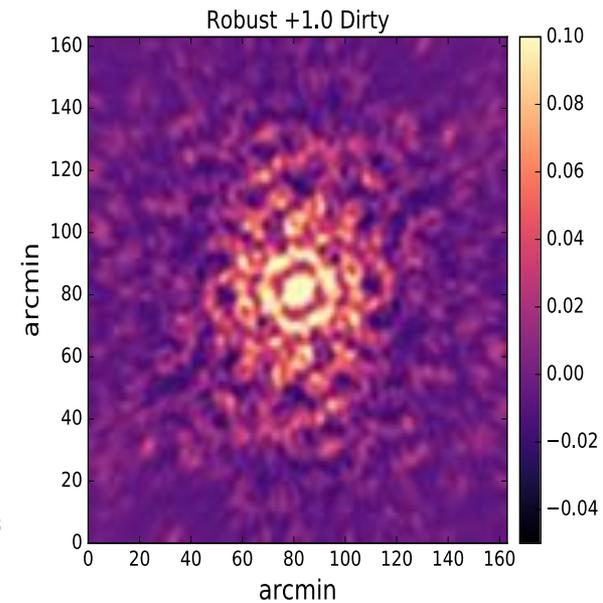
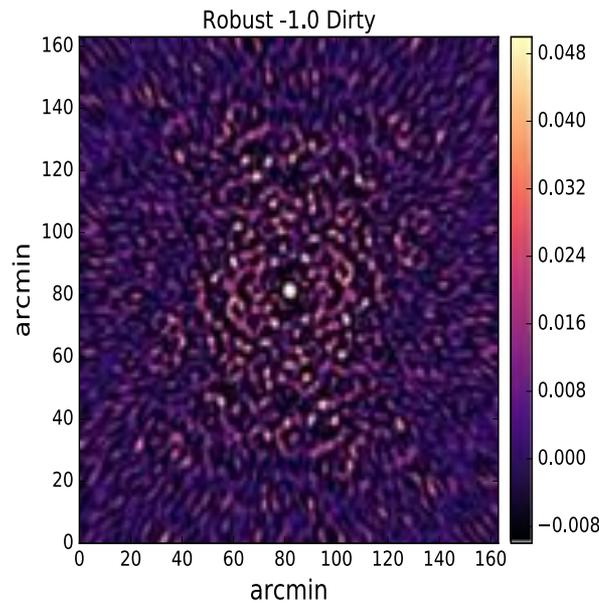


Issues – Dirty Beams

- Usually assume clean beam in P(D) calculation

→ very bad if clean and dirty beams very different !!!

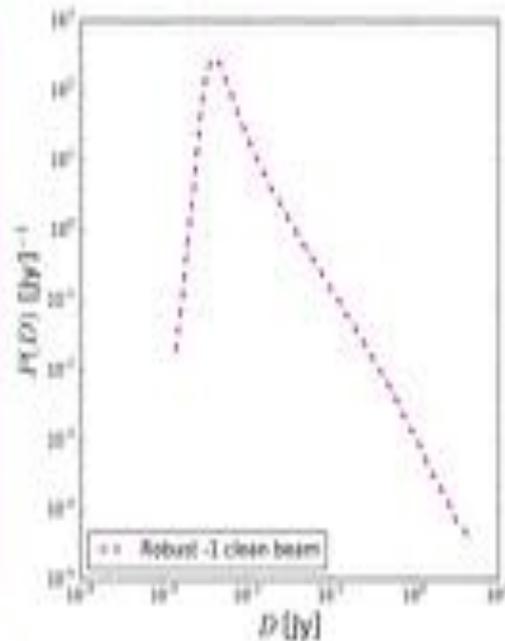
MWA Beams 2-minute snapshot →



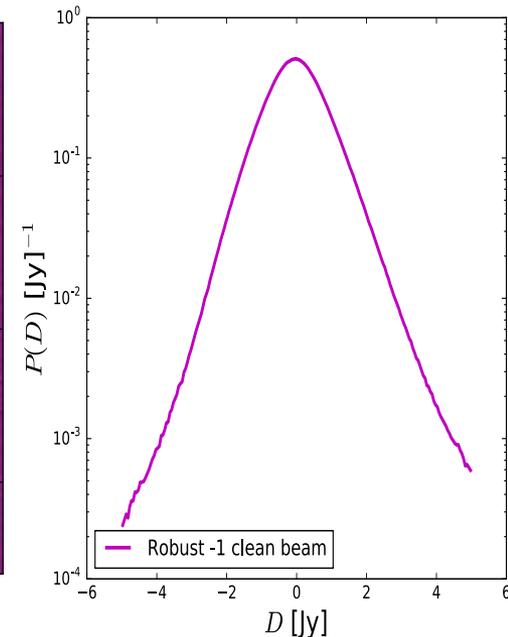
Issues – Dirty Beams

- Usually assume clean beam in $P(D)$ calculation = very bad if clean and dirty beams very different !!!
 - Can calculate $P(D)$ using just dirty beam or dirty beam for sources with $S < \text{cleaning limit}$ and clean beam for $S > \text{clean limit}$
 - If ignored and fitting source count could discover a whole new population of sources (which are really sidelobes)
 - This also allows for an estimate of the noise contribution from sidelobes

Robust -1 Clean beam



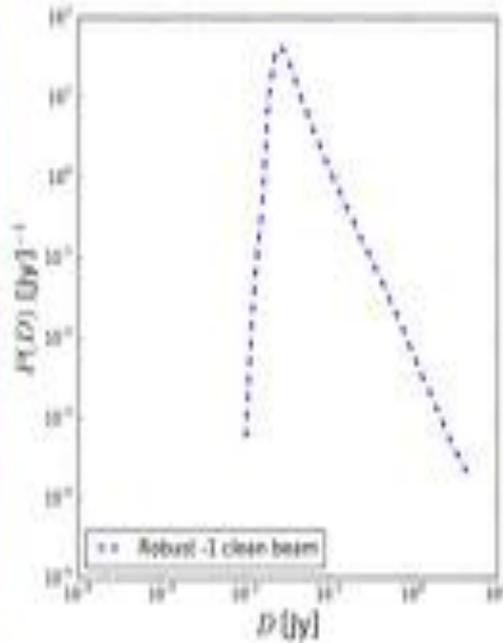
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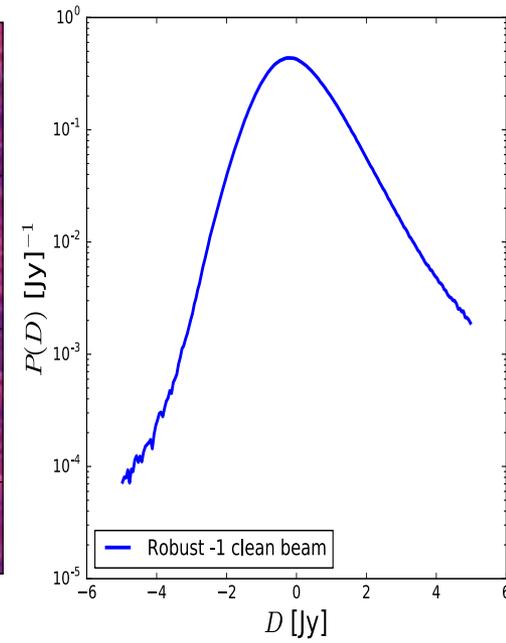
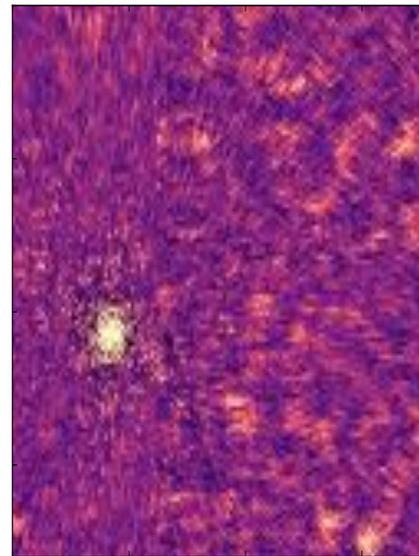
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Robust +1 Clean beam

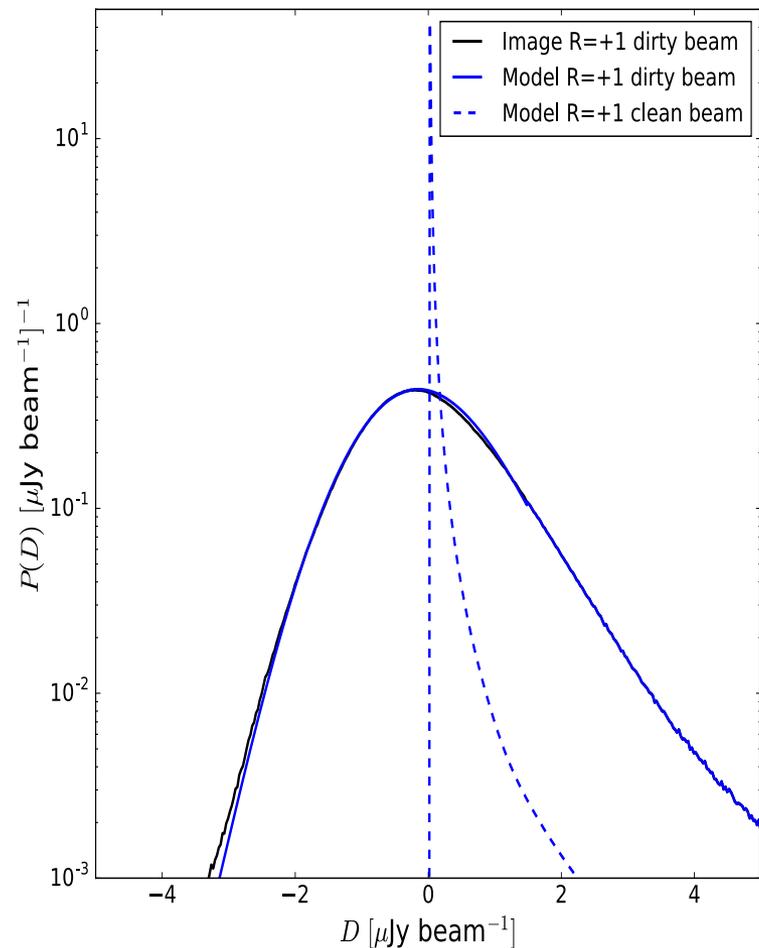


Robust +1 Dirty beam

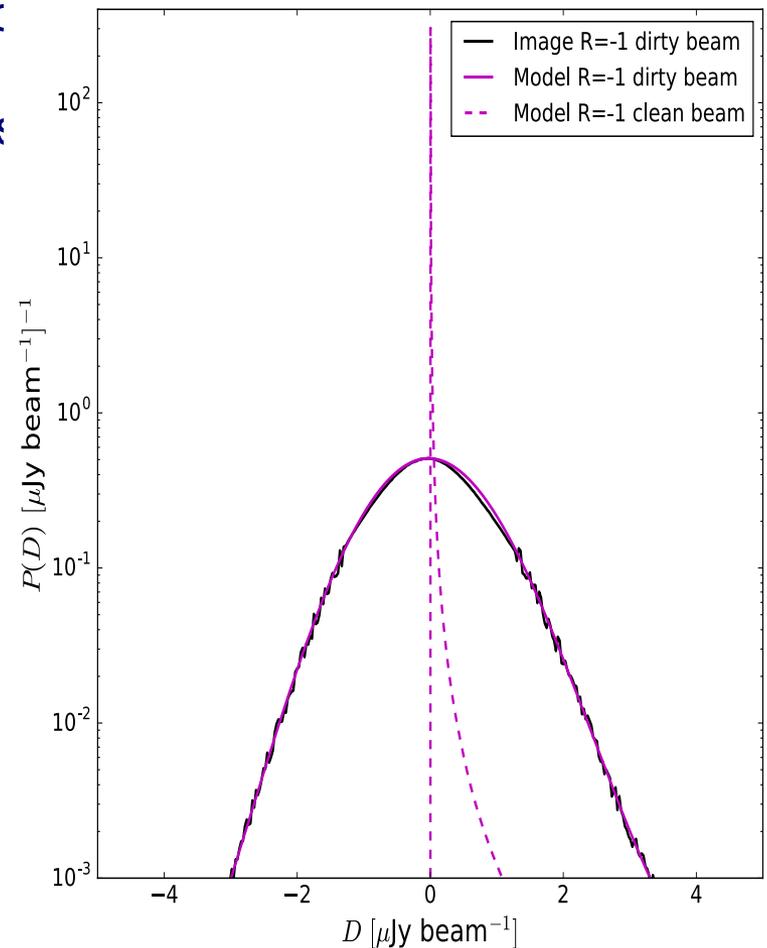


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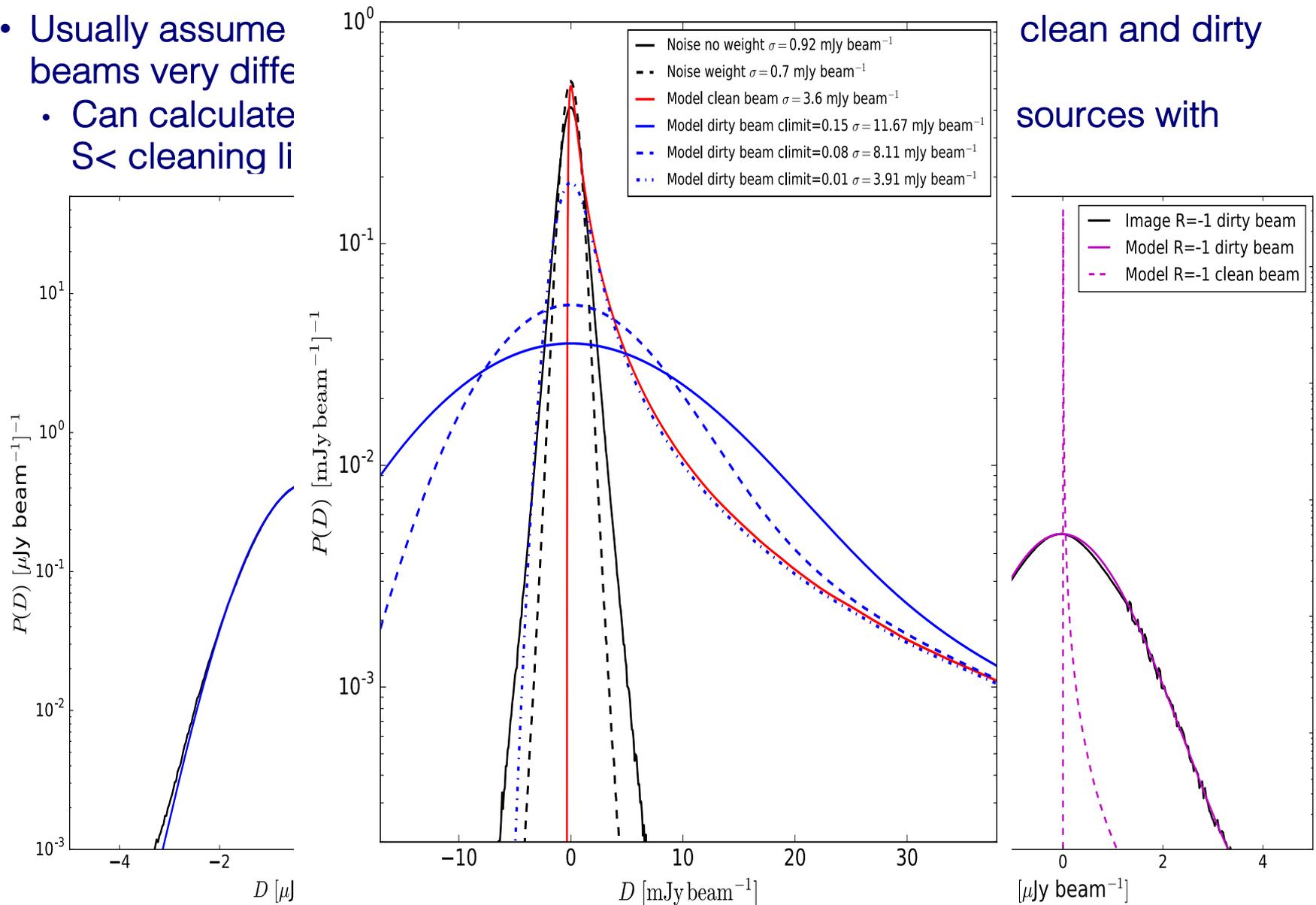


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Issues – Dirty Beams

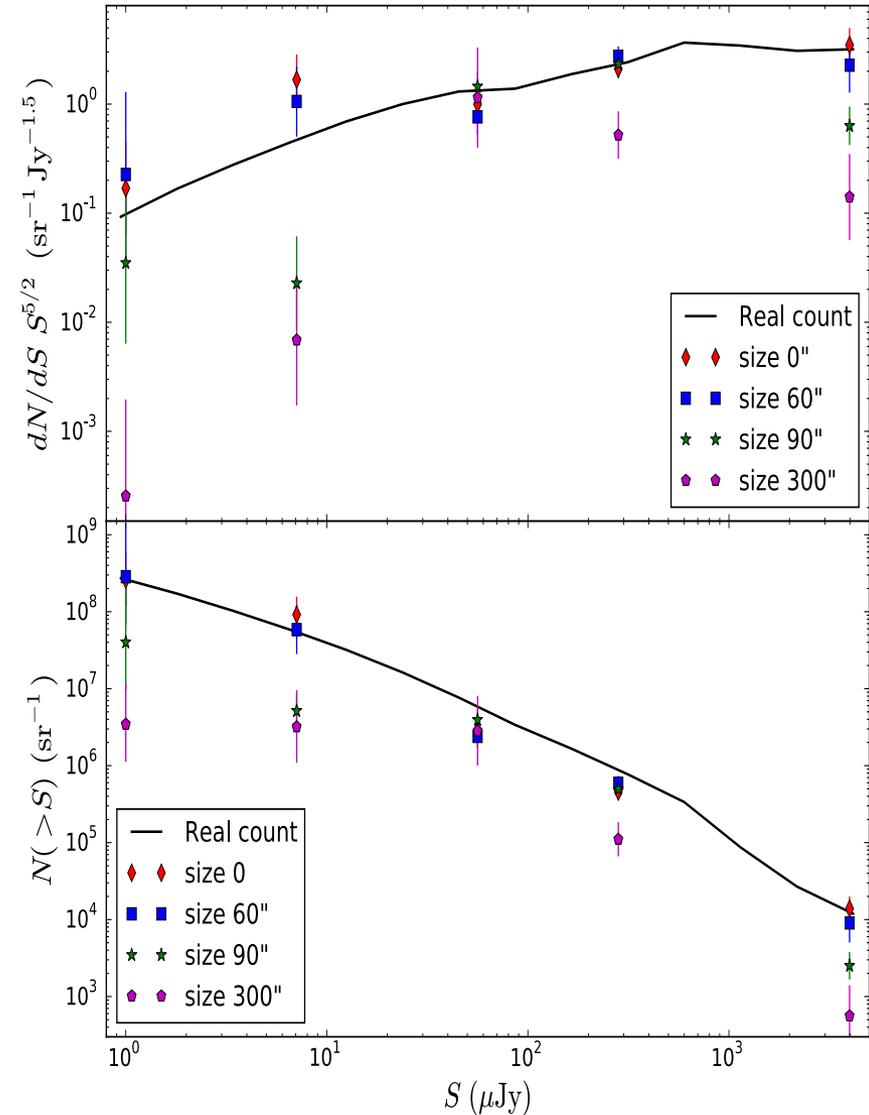
- Usually assume beams very diff
 - Can calculate $S < \text{cleaning li}$



clean and dirty
sources with

Issues – Source Sizes

- P(D) assumes point sources
 - So what happens with extended sources?
 - Simulation ATCA beam ($\sim 90''$):
 - Point sources
 - $60''$ sources
 - $90''$ sources
 - $300''$ sources
 - Fitting returns consistent results for sources with $\Omega_{\text{size}} \leq \Omega_{\text{beam}}$
 - Any larger source count is underestimated
- P(D) works as long as sources are roughly beam size or smaller



Conclusions

- Confusion analysis is important and can be a powerful tool
- Can be used:
 - to estimate discrete source count below confusion and instrumental noise levels
 - to estimate cosmic radio background temperature
 - to estimate the contribution from diffuse emission
 - to model source population luminosity functions and spectral indices
 - to look at the correlation between frequencies
 - to obtain confusion noise contribution (and sidelobe contribution) to total noise for cataloguing
 - to obtain Poisson contribution to angular power spectrum
 - Polarisation as well as total intensity
- Things to remember:
 - Want instrumental noise $<$ confusion noise
 - $P(D)$ assumes constant noise \rightarrow use noise zones and weighted histograms
 - Need to use the right beam shape in calculation
 - $P(D)$ assumes point sources \rightarrow need beam size such that source size \leq beam
 - Other issues to be aware of: beam changes from ionosphere, non-Gaussian noise and artifacts

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